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PROJECT ADMINISTRATION DATA SHEET

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ject No. A-3410 DATE 11/23/82

ject Director: Joe L. Sims School/Lab EML/RSD HO

nsor: U. S. Army Missile Command; Redstone Arsenal, AL 35898

e Agreement: Delivery Order No. 0061 under Contract No. DAAH01-81-D-A003

ard Period: From 11/5/82 To 3/9/83 (Performance) 5/9/83 (Reports)

nsor Amount: $35,000 Contracted through:

t Sharing: None GTRI/GIT

e: Component Model Base Flow Evaluation RDF-60

MINISTRATIVE DATA OCA Contact Linda H. Bowman x4820

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ense Priority Rating: DO-A2 under DMS Reg. 1 Security Classification: Unclassified

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GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION SHEET

Date 6/7/83

Project Title: Component Model Base Flow Evaluation

Project No: A-3410

Project Director: J. L. Sims

Sponsor: U. S. Army Missile Command

Effective Termination Date: 3/9/83

Clearance of Accounting Charges: 5/9/83

Grant/Contract Closeout Actions Remaining:

- [X] Final Invoice ~~and Closing Documents~~
- [ ] Final Fiscal Report
- [X] Final Report of Inventions (Send patent questionnaire)
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Monthly Technical Report No. 1
and
Monthly Cost and Performance Report No. 1

Report Period
November 5 through November 30, 1982

Report Prepared
December 15, 1982

COMPONENT MODEL BASE FLOW EVALUATION

J. L. Sims

Contract No. DAAH01-81-D-A003
Delivery Order No. 0061
EES Project A-3410

Effective Date: 11/05/82
Expiration Date: 03/09/83

Prepared for

Commander
U.S. Army Missile Command
Attn: DRSMI-IYBB/Koger
Redstone Arsenal, AL 35898

Prepared by

Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia 30332

WORK PERFORMED DURING THIS REPORTING PERIOD

The overall objective of this effort is to modify an existing computer program for the prediction of power-on base pressure to incorporate the effect of the approach boundary layer and the effect of an incomplete recompression criteria for the reattachment streamlines. A review of some of the available literature revealed that there are basically two methods for including the effect of the upstream boundary layers upon the viscous mixing layer of the base flow field. These methods are: (1) use an incomplete shear layer velocity profile developing from the boundary layer velocity profile at the point of separation; and (2) use a complete shear layer velocity developing from an upstream origin, ahead of the point of separation, which is based upon the boundary layer momentum thickness at the point of separation. Results available in the literature show that both of these approaches are technically sound.

An indepth review of the computer program which is to be modified was completed. This review showed the computer program to be substantially more compatible with the origin shift, fully developed velocity profile method. Therefore, this method was chosen for implementation. Derivations of the origin shift and the mixing equations for this method have been completed using procuedres similar to those reported in MICOM Technical Report No. RD-TN-69-7. These equations have been specialized to the two stream axi-symmetric base pressure program to pinpoint the specific equation changes which must be incorporated into the computer program.

The literature review revealed a number of completely different techniques for calculating the incomplete recompression of the reattachment streamlines. These techniques vary from empirical, based on boundary layer separation data, to analytical, based on a control volume analysis. Since this effort can be implemented independent of the boundary layer effects, a decision on a technique is deferred until later.

PROBLEMS ENCOUNTERED

None

WORK PLANNED

The effort to incorporate the boundary layer effects into the existing computer program will be started. This work will be done on the version of the program which runs on the Perkin-Elmer 3220. Program changes will be made such that the program will run as presently structured or with consideration of the boundary layer effects. Further analytical work on the recompression criteria will be performed to form the basis for the selection of a particular criteria.

A-3410 COST INFORMATION

The following charges have been incurred against the contract during the period November 5 through November 30, 1982.

	Expended	Encumbered
Personal Services (PS)	$6,075.37	$ 0
Fringe Benefits	1,081.75	0
Materials and Supplies	0	0
Travel	0	0
Subtotal	$7,157.12	0
Equipment	0	0
Overhead (at 47.2% of Subtotal)	3,378.16	0
TOTAL	$10,535.28	0

The breakdown of personal services is as follows:

	Dollars	Approximate Man Hours
Principal Research Scientists/Engineers	$ 0	0
Senior Research Scientists/Engineers	4,648.71	203.0
Research Scientists II/Engineers II	1,426.66	78.0
Research Scientists I/Engineers I	0	0
Technicians/Draftsmen	0	0
Students	0	0
Secretarial/Clerical/Other	0	0
TOTAL	$6,075.37	281.0

The current financial status of the contract is as follows:

	Budget as Proposed	Expended	Encumbered	Free Balance
Personal Services	$18,704.00	$ 6,075.37	$ 0	$12,628.63
Fringe Benefits	3,928.00	1,081.75	0	2,846.25
Materials and Supplies	126.00	0	0	126.00
Travel	1,019.00	0	0	1,019.00
Equipment	0	0	0	0
Overhead	11,223.00	3,378.16	0	7,844.84
FUNDING	$35,000.00	$10,535.28	$ 0	$24,464.72

Based on present full funding, the funding and equivalent man hours are sufficient to complete the task. Approximately 30% of the proposed task has been completed.

Monthly Technical Report No. 2
and
Monthly Cost and Performance Report No. 2

Report Period
December 1 through December 31, 1982

Report Prepared
January 18, 1983

COMPONENT MODEL BASE FLOW EVALUATION

J. L. Sims

Contract No. DAAH01-81-D-A003
Delivery Order No. 0061
Project No. A-3410

Effective Date: 11/05/82
Expiration Date: 03/09/83

Prepared for

U. S. Army Missile Command
Attn: DRSMI-IYBB/Koger
Redstone Arsenal, Alabama 35898

Prepared by

Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia 30332

WORK PERFORMED DURING THIS REPORTING PERIOD

A set of equations defining the effect of the upstream boundary layers upon the base pressure was completed in the previous reporting period. An analysis of these equations in consideration of the structure of the computer program was performed to determine the most efficient way to incorporate the required additions to the program. A preliminary plan has been developed for these program changes. These changes will incorporate: (1) the effect of the expansion at the base upon the upstream boundary layers; and (2) the effect of these boundary layers upon the mixing process. Priority consideration is being given to maintaining the program clarity and usability so as to maintain it as a viable, useful design tool.

Equation and solution techniques for a strong shock slipline solution were derived and formulated. This solution will extend the ability of the overall base pressure program to obtain solutions at somewhat lower free stream Mach numbers.

Analysis on the various recompression models was continued. A listing of a computer program, developed by Wagner and White, which implements the ONERA recompression criteria, was obtained from Dr. White at the University of Illinios. This code was analyzed to ascertain the method of application of the ONERA recompression criteria. It is anticipated that considerably more analysis will have to be performed before the currently used recompression criteria will be replaced.

PROBLEMS ENCOUNTERED

None

WORK PLANNED

Some time will be devoted to becoming familiar with the Perkin-Elmer OS/32 operating system and the performance of the base pressure computer program. Some critical case solutions will be sought to determine if program modifications are needed to improve the performance of this version of the program. If it is deemed necessary to modify the program, this effort will be initiated. The strong shock slipline solution will be coded, checked out and incorporated into the computer program. Performance of the program with the new slipline solution will be evaluated by re-running the critical case solutions. Work will continue to include the coding for the boundary layer effects in the overall program.

A-3410 COST INFORMATION

The following charges have been incurred against the contract during the period December 1 through December 31, 1982.

	Expended	Encumbered
Personal Services (PS)	$5,108.95	$ 0.00
Fringe Benefits	927.46	0.00
Materials and Supplies	0.00	0.00
Travel	0.00	0.00
Subtotal	$6,036.41	$ 0.00
Equipment	0.00	0.00
Overhead (at 47.2% of Subtotal)	2,849.19	0.00
TOTAL	$8,885.60	$ 0.00

The breakdown of personal services is as follows:

	Dollars	Approximate Manhours
Principal Research Scientists/Engineers	$ 0.00	0.0
Senior Research Scientists/Engineers	4,146.25	181.0
Research Scientists II/Engineers II	0.00	0.0
Research Scientists I/Engineers I	962.70	65.0
Technicians/Draftsmen	0.00	0.0
Students	0.00	0.0
Secretarial/Clerical/Other	0.00	0.0
TOTAL	$5,108.95	246.0

The current financial status of the contract is as follows:

	Budget As Proposed	Expended	Encumbered	Free Balance
Personal Services	$18,704.00	$11,184.32	$ 0.00	$7,519.68
Fringe Benefits	3,928.00	2,009.21	0.00	1,918.79
Materials and Supplies	126.00	0.00	0.00	126.00
Travel	1,019.00	0.00	0.00	1,019.00
Equipment	0.00	0.00	0.00	0.00
Overhead	11,223.00	6,227.35	0.00	4,995.65
FUNDING	$35,000.00	$19,420.88	$ 0.00	$15,579.12

Based on present full funding, the funding and equivalent manhours are sufficient to complete the task. Approximately 55% of the proposed task has been completed.

Monthly Technical Report No. 3
and
Monthly Cost and Performance Report No. 3

Report Period
January 1 through January 31, 1983

Report Prepared
February 16, 1983

COMPONENT MODEL BASE FLOW EVALUATION

J. L. Sims

Contract No. DAAH01-81-D-A003
Delivery Order No. 0061
Project No. A-3410

Effective Date: 11/05/82
Expiration Date: 03/09/83

Prepared for

U. S. Army Missile Command
Attn: DRSMI-IYBB/Koger
Redstone Arsenal, Alabama 35898

Prepared by

Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia 30332

WORK PERFORMED DURING THIS REPORTING PERIOD

A number of minor changes were made in the Perkin-Elmer (P-E) OS/32 version of the base pressure program (TSABPII) to account for the slight differences between this computer and the one for which the program was originally written. After these modifications were completed the program was checked by comparing results for selected non-critical cases with results which had been obtained on other computers. The agreement of these two sets of results was excellent. Next some critical cases (cases nearing the limit for which a solution can be found) were computed and compared to results from the CYBER. The results were (1) the P-E solved every case that the CYBER solved, and (2) the agreement of the two sets of solutions was excellent. Therefore, the basic program was judged to be satisfactorally operating on the P-E.

The strong shock slipline solution equations were coded as a subroutine, fully compatible with the base pressure program, and a short driver program was written for coding checkout. After a complete checkout, this new subroutine was inserted into the program replacing the old slipline subroutine. The base pressure program is now operational with the new slipline solution and the critical case solutions have been repeated. The results of these calculations are (1) the range of the available solutions for the critical cases has been extended, and (2) the desired tolerance on the slipline angle is always met and usually requires fewer iterations than the old subroutine.

Analysis to define the best way to include the coding for the boundary layer effects equations into the overall program was continued.

PROBLEMS ENCOUNTERED

None

WORK PLANNED

The plan for including the boundary layer effects in the overall program will be completed. Coding to implement this plan will be developed and inserted into the program and numerical checkout of the results will be performed. When the checkout is satisfactorally completed, cases will be run to compare with the calculations of Wagner and White. Analysis of the various recompression models will be continued as time permits.

A-3410 COST INFORMATION

The following charges have been incurred against the contract during the period January 1 through January 31, 1983.

	Expended	Encumbered
Personal Services (PS)	$4,118.96	$ 0.00
Fringe Benefits	778.02	0.00
Materials and Supplies	6.40	0.00
Travel	0.00	0.00
Subtotal	$4,903.38	$ 0.00
Equipment	0.00	0.00
Overhead (at 47.2% of Subtotal)	2,314.40	0.00
TOTAL	$7,217.78	$ 0.00

The breakdown of personal services is as follows:

	Dollars	Approximate Manhours
Principal Research Scientists/Engineers	$ 0.00	0.0
Senior Research Scientists/Engineers	3,980.40	172.0
Research Scientists II/Engineers II	0.00	0.0
Research Scientists I/Engineers I	0.00	0.0
Technicians/Draftsmen	0.00	0.0
Students	92.00	16.0
Secretarial/Clerical/Other	46.56	6.0
TOTAL	$4,118.96	194.0

The current financial status of the contract is as follows:

	Budget As Proposed	Expended	Encumbered	Free Balance
Personal Services	$18,704.00	$15,303.28	$ 0.00	$3,400.72
Fringe Benefits	3,928.00	2,787.23	0.00	1,140.77
Materials and Supplies	126.00	6.40	0.00	119.60
Travel	1,019.00	0.00	0.00	1,019.00
Equipment	0.00	0.00	0.00	0.00
Overhead	11,223.00	8,541.75	0.00	2,681.25
FUNDING	$35,000.00	$26,638.66	$ 0.00	$8,361.34

Based on present full funding, the funding and equivalent manhours are sufficient to complete the task. Approximately 76% of the proposed task has been completed.

Monthly Technical Report No. 4
and
Monthly Cost and Performance Report No. 4

Report Period
February 1 through February 28, 1983

Report Prepared
March 30, 1983

COMPONENT MODEL BASE FLOW EVALUATION

J. L. Sims

Contract No. DAAH01-81-D-A003
Delivery Order No. 0061
Project No. A-3410

Effective Date: 11/05/82
Expiration Date: 03/09/83

Prepared for

U. S. Army Missile Command
Attn: DRSMI-ICDB/Mulder
Redstone Arsenal, Alabama 35898

Prepared by

Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia 30332

WORK PERFORMED DURING THIS REPORTING PERIOD

The plan for including the boundary layer effects into the overall base pressure program was developed. This plan was developed to minimize the number of changes to the program and to make these changes in a coherent manner to maintain program clarity. Coding was developed and inserted into the program to incorporate the boundary layer effects calculations. These changes were thoroughly checked out by performing hand calculations to verify the program results from the changes.

After the program checkout, sample calculations were performed for the cases presented by Wagner and White in Vol 18, No. 8 of the AIAA Journal. The comparison of the two sets of calculations were very favorable. Therefore, it is felt that the calculations including the boundary layer effects are being done correctly with the current empirical recompression coefficient.

PROBLEMS ENCOUNTERED

None

WORK PLANNED

The final technical report will be written.

A-3410 COST INFORMATION

The following charges have been incurred against the contract during the period February 1 through February 28, 1983.

	Expended	Encumbered
Personal Services (PS)	$4,385.19	$ 0.00
Fringe Benefits	811.49	0.00
Materials and Supplies	7.20	0.00
Travel	0.00	0.00
Subtotal	$5,203.88	$ 0.00
Equipment	0.00	0.00
Overhead (at 47.2% of Subtotal)	2,456.23	0.00
TOTAL	$7,660.11	$ 0.00

The breakdown of personal services is as follows:

	Dollars	Approximate Manhours
Principal Research Scientists/Engineers	$ 0.00	0.0
Senior Research Scientists/Engineers	4,146.25	179.0
Research Scientists II/Engineers II	0.00	0.0
Research Scientists I/Engineers I	0.00	0.0
Technicians/Draftsmen	0.00	0.0
Students	212.75	37.0
Secretarial/Clerical/Other	26.19	3.0
TOTAL	$4,385.19	219.0

The current financial status of the contract is as follows:

	Budget As Proposed	Expended	Encumbered	Free Balance
Personal Services	$19,546.00	$19,688.47	$ 0.00	$-142.47
Fringe Benefits	4,104.00	3,598.72	0.00	505.28
Materials and Supplies	126.00	13.60	0.00	112.40
Travel	0.00	0.00	0.00	0.00
Equipment	0.00	0.00	0.00	0.00
Overhead	11,224.00	10,997.98	0.00	226.02
FUNDING	$35,000.00	$34,298.77	$ 0.00	$ 701.23

Based on present full funding, the funding and equivalent manhours are sufficient to complete the task. Approximately 98% of the proposed task has been completed.

FINAL REPORT
PROJECT A-3410

COMPONENT MODEL BASE FLOW EVALUATION

By

J. L. Sims

Prepared for

U. S. ARMY MISSILE COMMAND
Attn: DRSMI-ICDB/MULDER
REDSTONE ARSENAL, ALABAMA 35898

Under

Contract No. DAAH01-81-D-A003
Delivery Order No. 0061

Report Period from November 5, 1982 through March 9, 1983

GEORGIA INSTITUTE OF TECHNOLOGY
A Unit of the University System of Georgia
Engineering Experiment Station
Atlanta, Georgia 30332

1983

FINAL TECHNICAL REPORT

Report Period
November 5, 1982 through March 9, 1983

Report Prepared
April 1983

COMPONENT MODEL BASE FLOW EVALUATION

J. L. Sims

Contract No. DAAH01-D-81-A003
Delivery Order No. 0061
EES Project A-3410

Prepared for

U.S. Army Missile Command
Redstone Arsenal, Alabama 35898

Prepared by

Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia 30332

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
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4. TITLE (and Subtitle) COMPONENT MODEL BASE FLOW EVALUATION		5. TYPE OF REPORT & PERIOD COVERED Final Technical Report 11/5/82 thru 3/9/83
		6. PERFORMING ORG. REPORT NUMBER A-3410-001
7. AUTHOR(s) Joseph L. Sims		8. CONTRACT OR GRANT NUMBER(s) DAAH01-81-D-A003 Delivery Order 0061
9. PERFORMING ORGANIZATION NAME AND ADDRESS Georgia Institute of Technology Engineering Experiment Station Atlanta, Georgia 30332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
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16. DISTRIBUTION STATEMENT (of this Report)

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Power-On Base Pressure
Base Pressure
Component Model Base Pressure

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report was prepared by the Electromagnetics Laboratory of the Engineering Experiment Station, Georgia Institute of Technology, under contract DAAH01-81-D-A003. Analyses were performed and computer program modifications were made to include (1) a strong shock solution for the slip streamline calculation, and (2) upstream boundary layer effects, in the calculations of power on base pressure. The basic calculation procedure uses a Korst type component model and the boundary layer at the base is represented by a free shear

(Continued)

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20. ABSTRACT (Continued)

layer with an effective origin shifted upstream of the base. Calculations were performed for a limited number of cases. These results were compared with both previous calculations and experimental data. These comparisons are presented in the report.

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LIST OF FIGURES

I. INTRODUCTION

The U.S. Army Missile Command sponsored the development of a computer program for the solution of missile power-on base pressure. This computer program, documented in References 1 and 2 was developed using the component flow model of Korst, et. al. One of the critical elements of this flow model was the so called recompression criterion which defined the dividing or stagnating streamlines at the intersection of the free stream and nozzle plume boundary streamlines. Reference 3 presented a comparison of experiments and theory for cylindrical afterbodies and developed an empirical correlation equation for a recompression coefficient which is applicable only to bodies with cylindrical afterbodies. The basic computer program was extended to account for boattailed or flared afterbodies by Reference 2 and the empirical equation for the cylindrical afterbody recompression coefficient was also incorporated into the program.

The programs were originally written in FORTRAN IV for the IBM 7094 and the IBM 360/75 systems and were later transferred to a CYBER 175 and then to a Perkin-Elmer OS32 and 3230. A large number of cases was run on the IBM machines and the program was found to be very reliable, within certain definable operating limits. Operations on the CYBER were equally satisfactory but the operations on the Perkin-Elmer machines were not. There is a difference in word length between the CYBER (60 bit) and the Perkin-Elmer (32 bit) and the program was switched from batch mode input for the CYBER to an interactive mode input for the Perkin-Elmer. It was thought that the less than completely satisfactory operation on the Perkin-Elmer might be caused by the shorter word length conflicting with some rather small tolerances in the various iteration procedures used in the program.

The work reported herein was divided into three major areas. First, the Perkin-Elmer version of the program was to be thoroughly reviewed and program changes instituted to maximize its performance with respect to the CYBER results. Second, the original slipline subroutine, which allowed only a weak shock solution, was to be replaced by a new slipline subroutine which included a strong shock solution, as required, to extend the Mach number or pressure ratio for which a solution can be found. Third, the effects of the upstream boundary layers of both the freestream and the nozzle flow were to be included in the governing mixing equations.

The period of performance for this investigation was November 5, 1982 through March 9, 1983.

II. PROGRAM ADAPTION TO PERKIN-ELMER 3230

The desired mode of operation of the program on the Perkin-Elmer 3230 is interactive with input from a terminal whereas the IBM and CYBER versions operate in batch mode with card deck input. An interactive version of the program with screen cues for input data was available at the time of task initiation. An examination of the FORTRAN listing revealed that some of the input data cues did not agree with the requirements established by the program logic. Upon compilation, FORTRAN errors were found in one subroutine of the program. These errors were found to arise from a difference in the way the Perkin-Elmer system handles dimensioned variables in statement functions within a subroutine. Minor programming changes corrected these error conditions and the input data cues were revised to correspond with the program requirements.

Calculations were performed for some of the cases which are presented in Reference 3. Comparisons of the two sets of results presented in Figures 1 through 6 show that the agreement between the results from the machines is outstanding. Thus, the difference in word lengths does not affect the solution accuracy within the resolution of the figures. The cases for the presentation of data in these figures are not "critical" cases, in that neither the Mach number nor the pressure ratio of these cases approaches the limit for which the program will not operate. A critical case solution from the CYBER was available as a standard of comparison and is presented in Table 1. The solution presented in Table 1 is for the lowest free stream Mach number for which the CYBER could obtain a solution. This critical case was also tried on the Perkin-Elmer 3230, with the solution presented in Table 2 for comparison with the CYBER solution. The difference in the base pressure ratio between the two solutions is only 8×10^{-5} which is less than the iteration tolerance in the base pressure iteration procedure. Therefore, it is concluded that the Perkin-Elmer version of the program satisfactorily duplicates results from the machines with greater accuracy in individual arithmetic operations. Furthermore, the word length of the Perkin-Elmer 3230 is sufficient, in general, to meet the iteration tolerances specified for the IBM and CYBER versions of the program. Thus, it was not necessary to increase the iteration tolerances to prevent the program from going to the maximum number of iterations before solutions were obtained.

In order to convert the program to the Perkin-Elmer, the Aerodynamics Branch of the U.S. Army Missile Command changed the batch mode Namelist input data to an interactive mode with the screen cues for each piece of input data. A complete set of screen cues, input data read statements, and input data definitions is given in Appendix A. The information in this appendix combined with the engineering data for the configuration and conditions should enable one to successfully run the Perkin-Elmer version of the base pressure program.

Output format of the Perkin-Elmer version of the program is exactly the same as that of the IBM and CYBER programs. In order for the user of the Perkin-Elmer version of the program to have a complete user's manual, definitions of the printed output data are presented in Appendix B for the cylindrical afterbody case. Output format for bodies with flared or boattailed afterbodies is the same as that of the cylindrical afterbody with some additional initial output of afterbody geometry data and flow field data. Tables 1 through 3 are replicas of the cylindrical afterbody output with error messages, which sometimes occur during intermediate iterations, deleted.

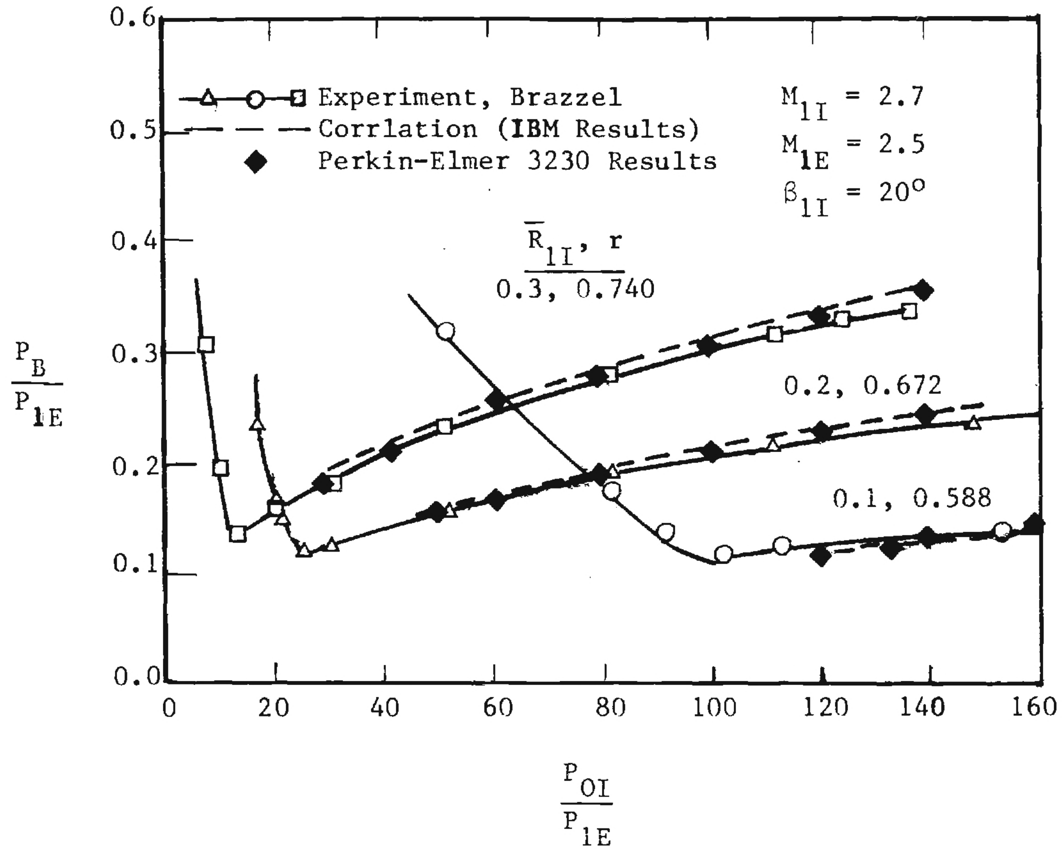

Figure 1. Comparison of Perkin-Elmer 3230 and IBM calculations for variations in nozzle radius (after Ref. 2, Fig. 16(C))

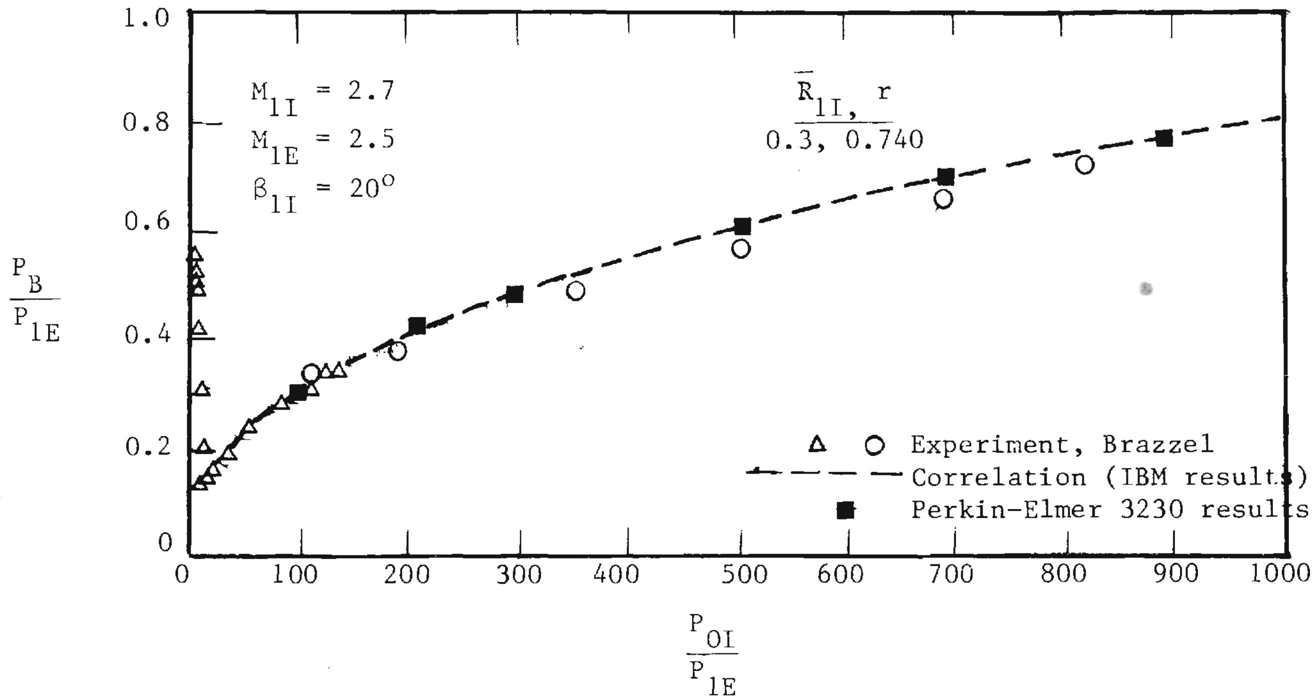

Figure 2. Comparison of Perkin-Elmer 3230 and IBM calculations for large range variations of pressure ratio (after Ref 2, Fig. 16D).

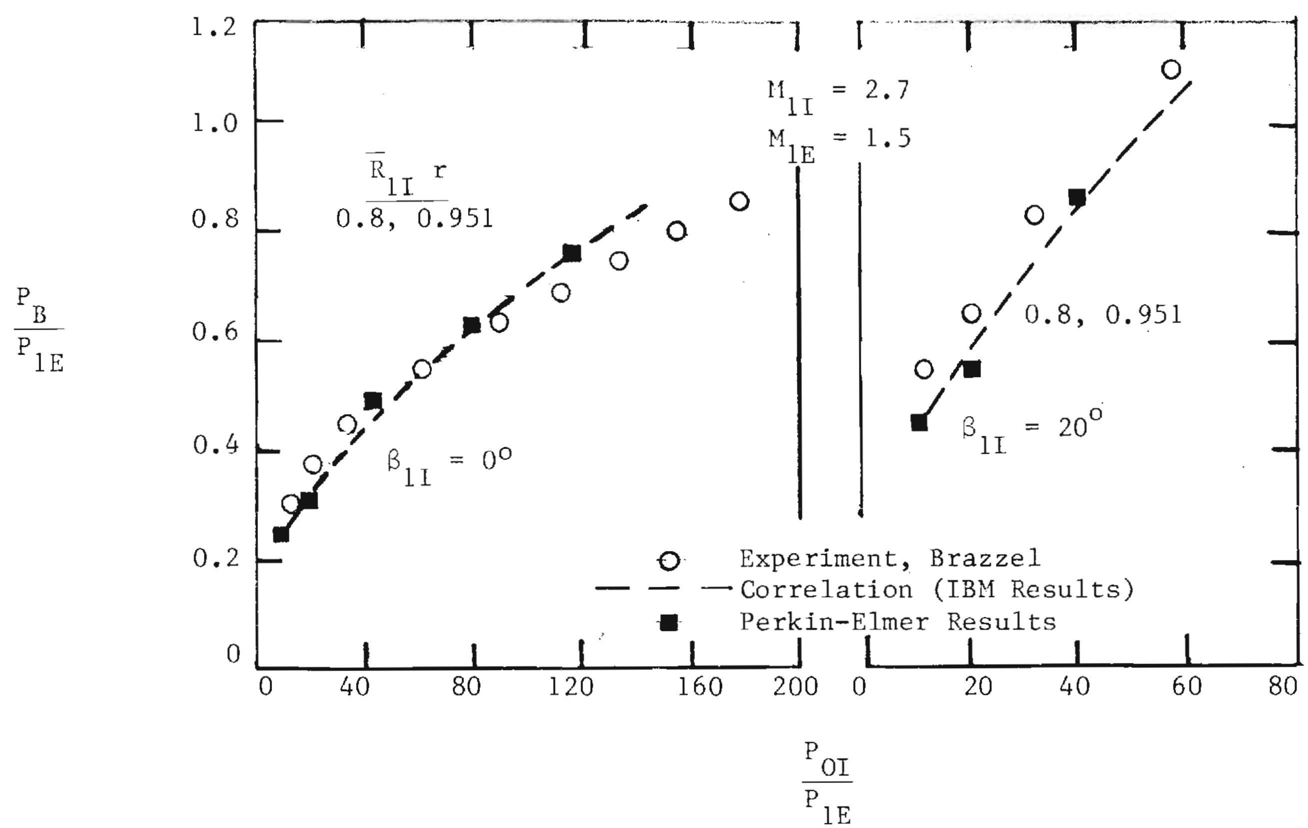

Figure 3. Comparison of Perkin-Elmer 3230 and IBM calculations for variations of nozzle wall angle at $M_\infty = 1.5$ (after Ref. 2, Fig. 16(i)).

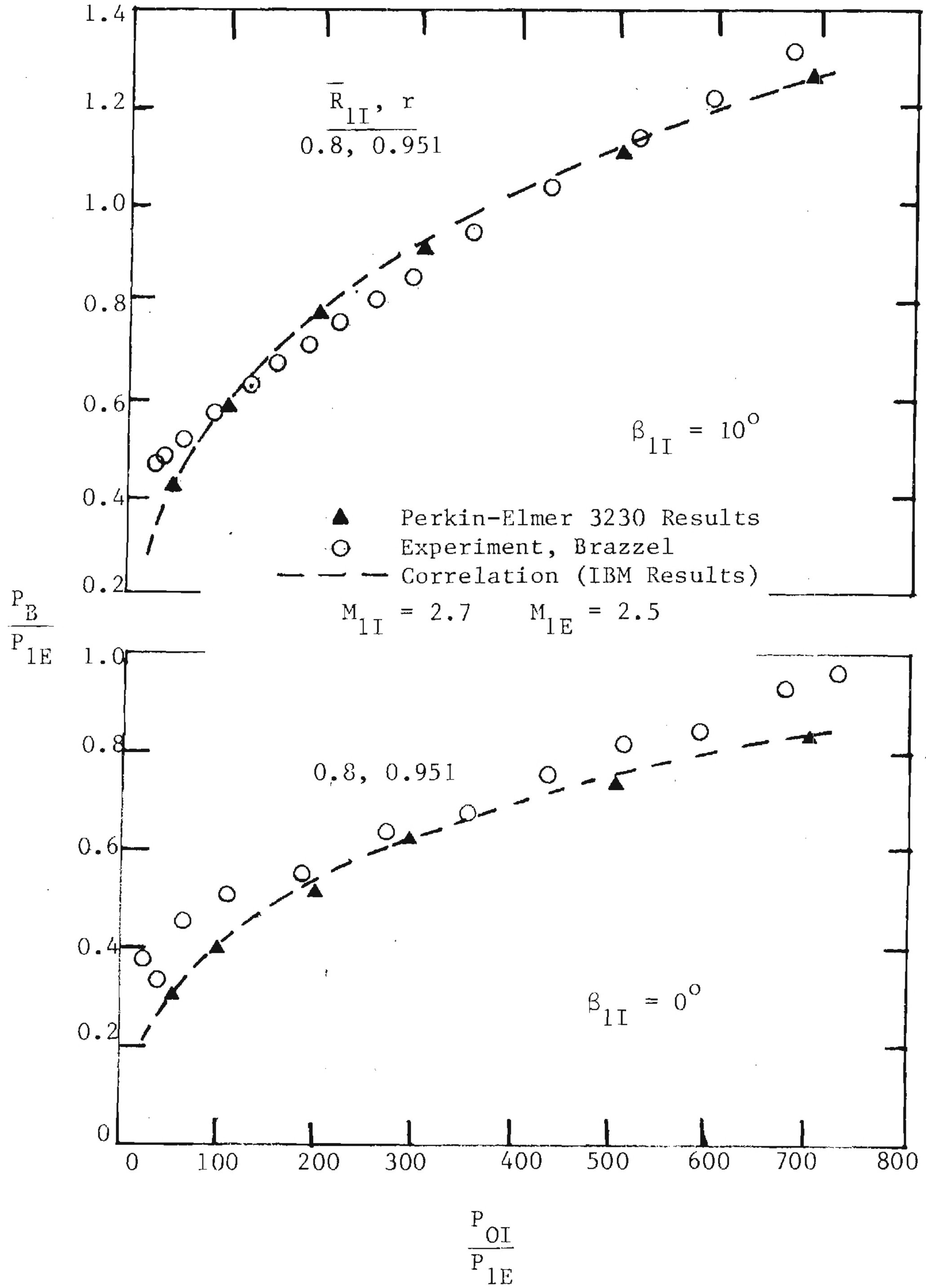

Figure 4. Comparison of Perkin-Elmer 3230 and IBM calculations for variations of nozzle wall angle at M_∞ = 2.5 (after Ref 2, Fig. 26 (j)).

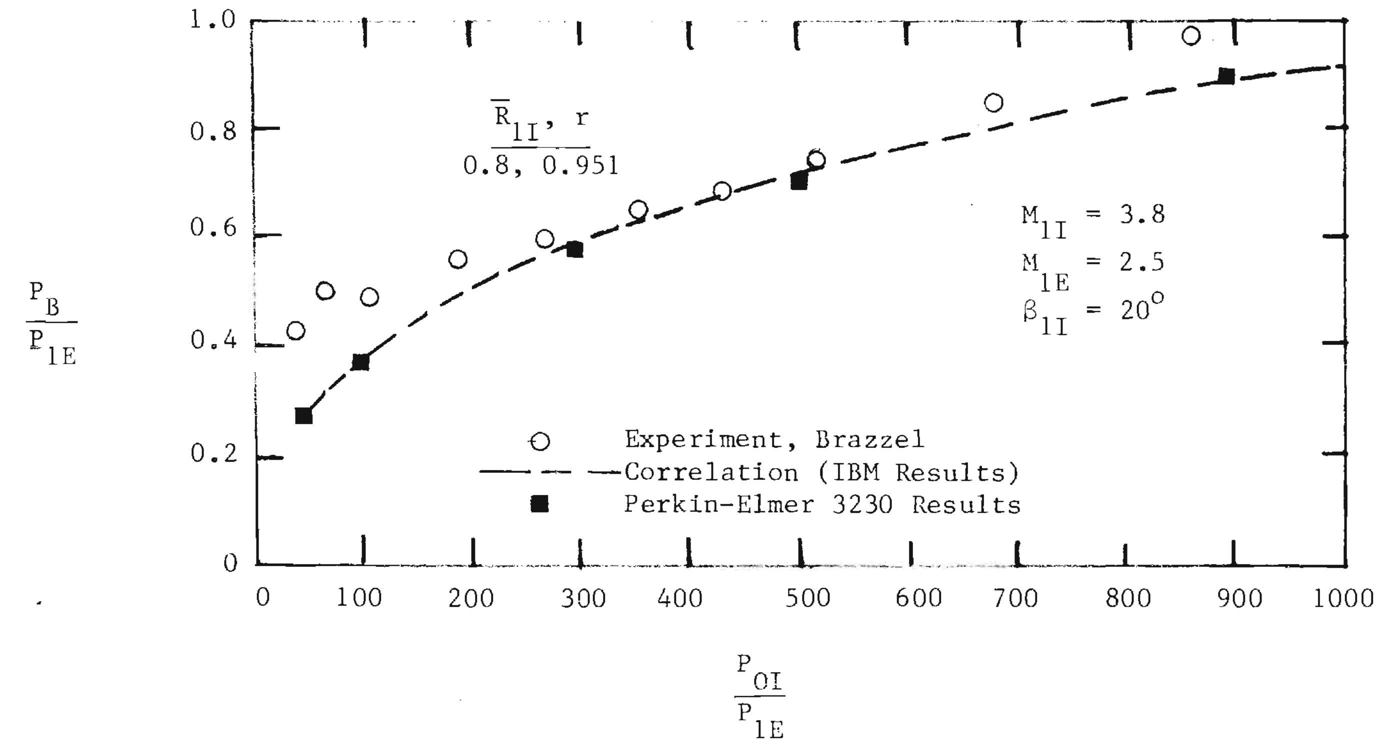

Figure 5. Comparison of Perkin-Elmer 3230 and IBM calculations for large range variation of pressure ratio at a large nozzle Mach number (after Ref 2, Fig. 16 (k)).

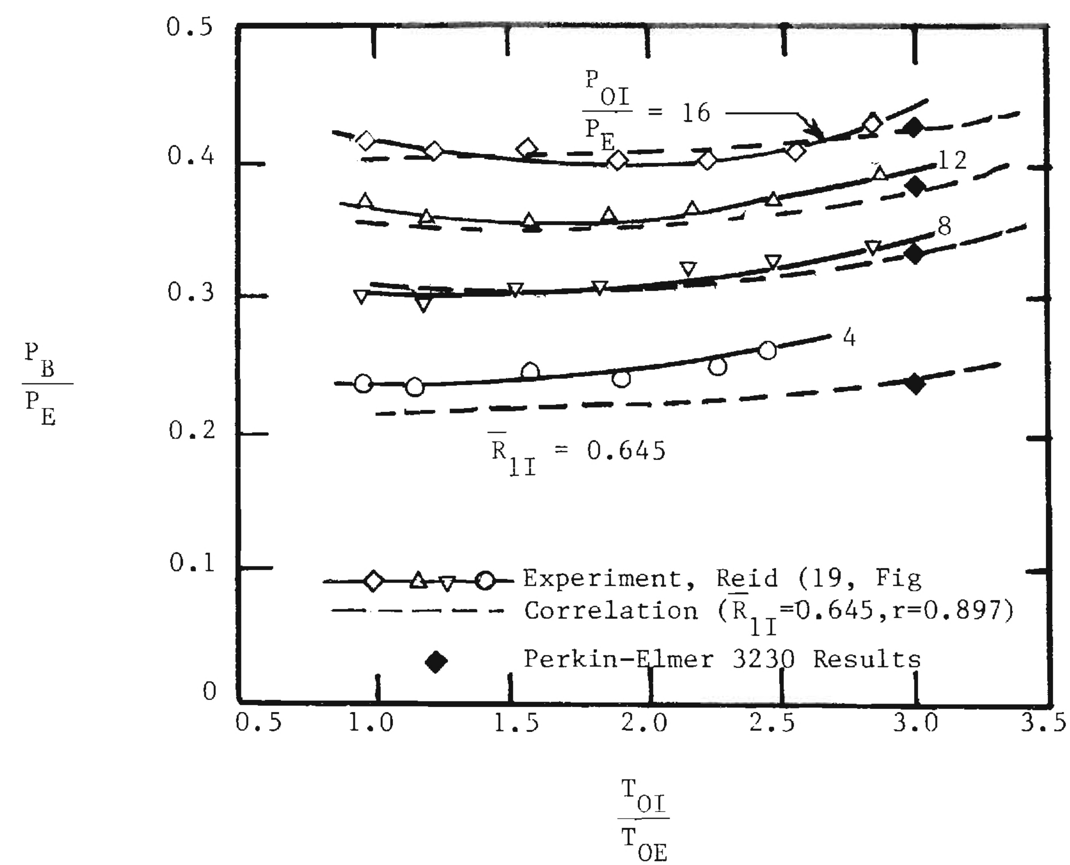

Figure 6. Comparison of Perkin-Elmer 3230 and IBM calculations for a temperature ratio of 3.0 (after Ref. 2, Fig. 15(b)).

TABLE 1. CRITICAL CASE CYBER SOLUTION

```
****TWO-STREAM BASE PRESSURE PROGRAM****                    PROB. NO.    1

                         ******INPUT DATA******

                         ***INTERNAL STREAM***

 GAMMAI= 1.400                 GAS CONSTANT  =      55.16 LB-FT/LB-R
X1I=  0.000                   R1I=    .250            BETA1I(DEG)=  10.000
EMN1I = 2.7000                EMS1I = 1.8865          P1I/POI = .04295

                         ***EXTERNAL STREAM***

 GAMMAE= 1.400                 GAS CONSTANT =      53.35 LB-FT/LB-R
X1E=  0.000                   R1E=  1.250             BETA1E(DEG)=   0.000
EMN1E = 1.4308                EMS1E = 1.3202          P1E/PO1E = .30084

                    ******BASE PRESSURE CASE DATA******

 P1I/PE =    2.1529                                   TOE/TOI =  1.00000
BLDRO =   0.                                         ENGRO = 0.

                  ***RECOMPRESSION COEFFICIENT = .672***
                ********************************************

                      ***ISOENERGETIC SOLUTION***
                    *********************************

                  ******TURBULENT JET MIXING RESULTS******

                           ***CURRENT DATA***

P1I/PE   =  2.15291                                 TOE/TOI   =  1.00000
POE/POI   =   .06631                                POI/PE    = 50.126
BLDRO =  0.                                         ENGRO   =   0.

                           ***MIXING DATA***

BLDR =   -.21803E-04                                ENGR   =   .41170E-03

                ***BASE PRESSURE AND TEMPERATURE RESULTS***

TB/TOE   =  1.00000                                 TB/TOI   =  1.00000
PB/PE    =   .31101                                 PB/P1I   =   .14446
CP-B     =  -.48079                                 CD-B     =   .46156

                 ******END OF CURRENT CASE RESULTS******
                *****************************************
```

TABLE 2. CRITICAL CASE PERKIN-ELMER 3230 SOLUTION

```
        ****TWO-STREAM BASE PRESSURE PROGRAM****      PROB. NO.      1
******INPUT DATA******

                        ***INTERNAL STREAM***

GAMMAI= 1.400                GAS CONSTANT  =     55.16 LB-FT/LB-R
X1I=  0.000                  R1I=  0.250            BETA1I(DEG)=  10.000
EMN1I = 2.7000               EMS1I = 1.8865         P1I/POI =0.04295

                        ***EXTERNAL STREAM***

GAMMAE= 1.400                GAS CONSTANT  =     53.35 LB-FT/LB-R
X1E=  0.000                  R1E=  1.250            BETA1E(DEG)=   0.000
EMN1E = 1.4308               EMS1E = 1.3202         P1E/PO1E = 0.30084

                   ******BASE PRESSURE CASE DATA******

P1I/PE   =  2.1529                                  TOE/TOI   =  1.00000
BLDRO   =  0.00000E+00                              ENGRO   = 0.00000E+00

                ***RECOMPRESSION COEFFICIENT = 0.672***
              ********************************************

                      ***ISOENERGETIC SOLUTION***
                  ********************************

                ******TURBULENT JET MIXING RESULTS******

                          ***CURRENT DATA***

P1I/PE   =  2.15291                                 TOE/TOI   =  1.00000
POE/POI =  0.06631                                  POI/PE    = 50.126
BLDRO    =  0.00000E+00                             ENGRO = 0.00000E+00

                          ***MIXING DATA***

BLDR = -0.15749E-04                                 ENGR = 0.41807E-03


              ***BASE PRESSURE AND TEMPERATURE RESULTS***

TB/TOE   =  1.00000                                 TB/TOI   =  1.00000
PB/PE    =  0.31109                                 PB/P1I   =  0.14450
CP-B     = -0.48074                                 CD-B     =  0.46151

               ******END OF CURRENT CASE RESULTS******
              ********************************************
```

TABLE 3. CRITICAL CASE PERKIN-ELMER 3230 SOLUTION
STRONG SHOCK SLIPLINE SUBROUTINE

```
     ****TWO-STREAM BASE PRESSURE PROGRAM****          PROB. NO.     1
******INPUT DATA******

                          ***INTERNAL STREAM***

GAMMAI= 1.400                 GAS CONSTANT =      55.16 LB-FT/LB-R
X1I=  0.000                   R1I=  0.250            BETA1I(DEG)=  10.000
EMN1I = 2.7000                EMS1I = 1.8865         P1I/POI = 0.04295

                          ***EXTERNAL STREAM***

GAMMAE= 1.400                 GAS CONSTANT =      53.35 LB-FT/LB-R
X1E= 0.000                    R1E= 1.250             BETA1E(DEG)=   0.000
EMN1E = 1.4308                EMS1E = 1.3202         P1E/P01E = 0.30084

                    ******BASE PRESSURE CASE DATA******
P1I/PE = 2.1529                                      TOE/TOI = 1.00000
BLDRO = 0.00000E+00                                  ENGRO = 0.00000E+00

                 ***RECOMPRESSION COEFFICIENT = 0.668***
             *********************************************


                       ***ISOENERGETIC SOLUTION***
                       ***************************

                 ******TURBULENT JET MIXING RESULTS******

                           ***CURRENT DATA***

P1I/PE   =   2.15291                                 TOE/TOI = 1.00000
POE/POI  =   0.06631                                 POI/PE  = 50.126
BLDRO    =   0.00000E+00                             ENGRO   = 0.00000E+00

                           ***MIXING DATA***

BLDR = -0.38926E-02                                  ENGR = -0.33739E-02

               ***BASE PRESSURE AND TEMPERATURE RESULTS***

TB/TOE =   1.00000                                   TB/TOI = 1.00000
PB/PE  =   0.31079                                   PB/P1I = 0.14436
CP-B   =  -0.48095                                   CD-B   = 0.46171

                 ******END OF CURRENT CASE RESULTS******
```

III. STRONG SHOCK SLIPLINE SOLUTION

One of the elements of the overall base pressure solution is the solution for the slip streamline (slipline) angle downstream of the shock wave systems existing at the intersection of the boundary streamlines. Figure 7 which is taken from Reference 1 depicts the situation for the "corresponding" inviscid flow field. Since the static pressure on both sides of the streamline must be equal for an equilibrium condition and P_B is known, the requirement is to find a pressure ratio, P_S/P_B, such that the flow boundary angles of both streams downstream of the shocks are equal. A schematic of the shock turning phenomena as applied to the two stream slipline process is shown in Figure 8. Referring to Figures 7 and 8, it is seen that the requirement for equal slipline angles from both streams is

$$\beta_{TOT} = \beta_{WS} + \beta_{SS} = \Delta\theta = \theta_{BI} - \theta_{BE} = \beta_I + \beta_E \qquad (1)$$

and a solution to the slipline angle exists only if $\Delta\theta$ is less than $\beta_{TOT_{MAX}}$. The slipline solution in the original version of the base pressure program was confined to the "weak" shock solution which limited the total deflection capability to the two stream deflection occurring at the pressure ratio where the weak stream reaches its maximum deflection. Since the strong shock solution is a physically possible solution extending the range for which solutions can be found, the program has been modified to include this capability.

The solution equations are independent of the relative strength of the two streams. However, the weak stream needs to be identified in the programming to provide a rational starting technique for the maximum deflection numerical solution iteration procedure. The deflection for either stream is defined by Reference 4, Equation 160:

$$\beta = \tan^{-1}\left\{\frac{\xi-1}{\gamma M^2 - \xi+1}\sqrt{\frac{2\gamma M^2 - (\gamma-1) - (\gamma+1)\xi}{(\gamma+1)\xi + (\gamma-1)}}\right\} \qquad (2)$$

where $\xi = P_S/P_B$ and the other variables are defined on the free streamline boundary upstream of the recompression shock. Now the required $\Delta\theta$ ($= \theta_{BI} - \theta_{BE}$) is known from the solution of the impingement point of the two streams. To determine if a solution to the slipline angle is possible, the maximum possible two-stream flow deflection, $\beta_{TOT_{MAX}}$, must be found and compared to the required total flow deflection, $\Delta\theta$. This is accomplished by differentiating Equation 1, setting the derivative equal to zero and solving the resultant equation for the pressure ratio. Thus,

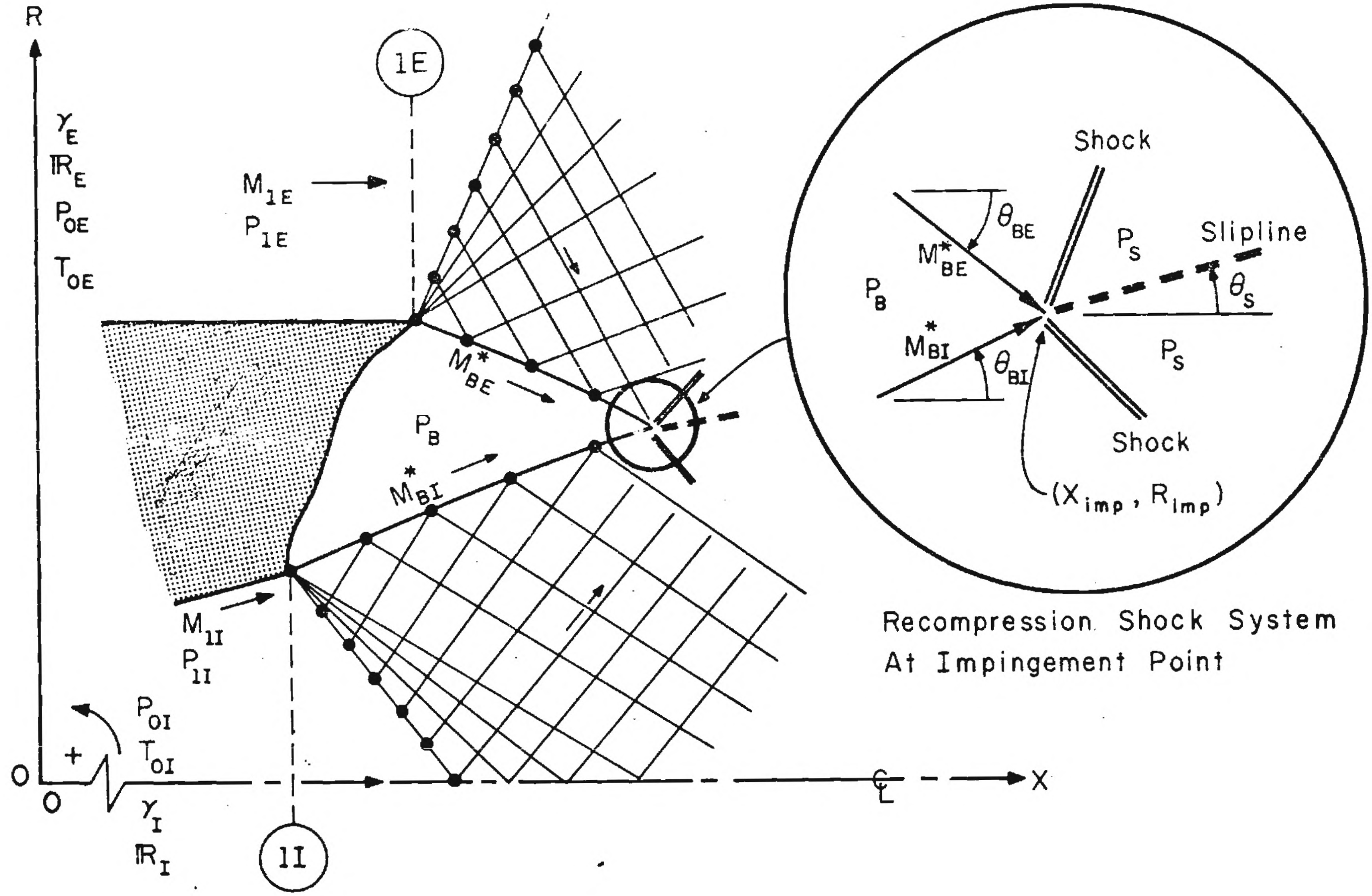

Figure 7. "Corresponding" Inviscid Flow Field.

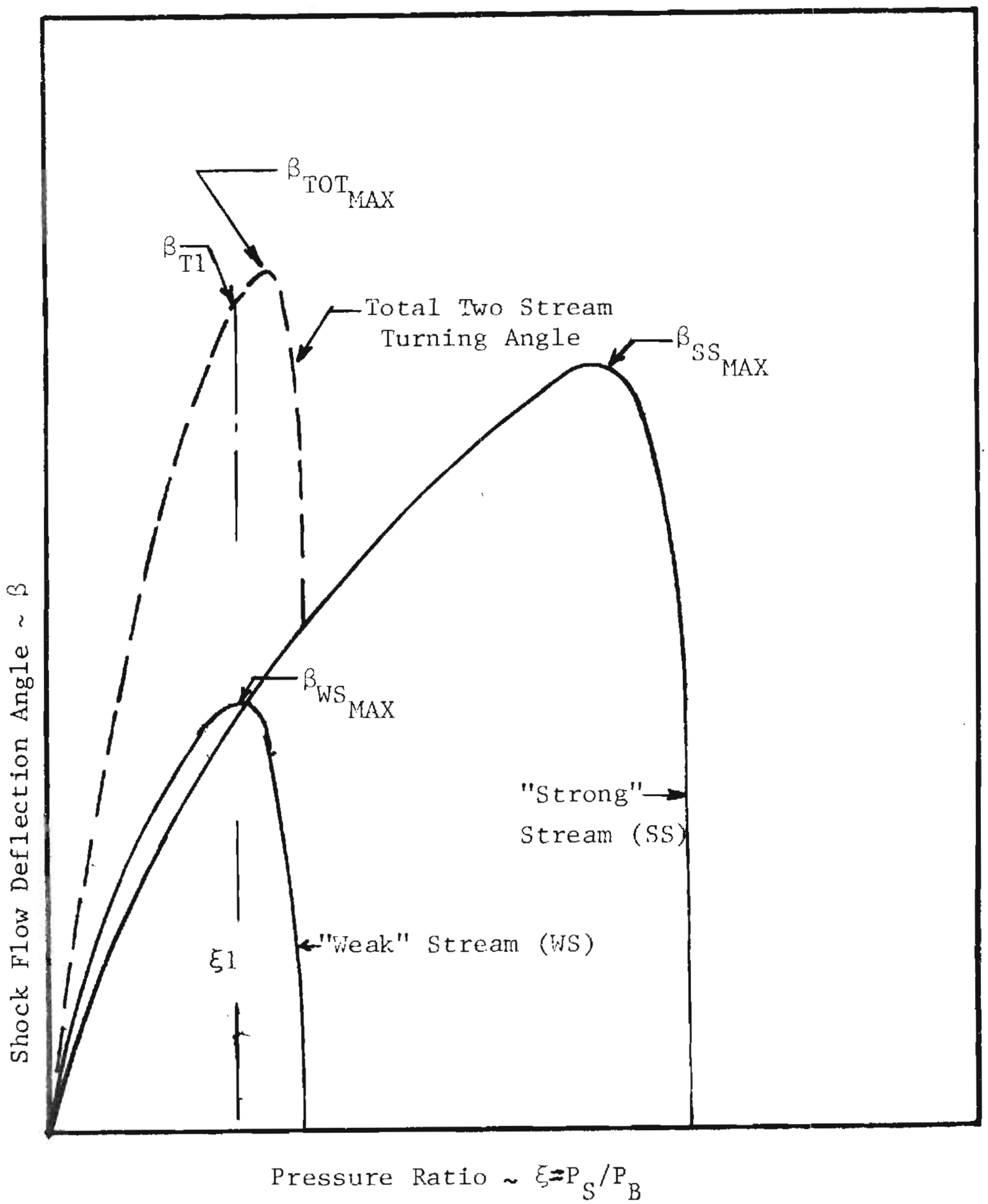

Figure 8. Shock turning angles for supersonic streams.

$$\beta_{TOT} = \beta_I + \beta_E \tag{3}$$

and

$$\frac{d\beta_{TOT}}{d\xi} = 0 = \frac{d\beta_I}{d\xi} + \frac{d\beta_E}{d\xi} \tag{4}$$

When Equation 2 is differentiated, there results

$$\frac{d\beta}{d\xi} = \left(\frac{1}{1+u^2}\right)\left(\frac{\gamma M^2}{\gamma M^2-\xi+1}\right) \sqrt{\frac{(\gamma+1)\xi + (\gamma-1)}{2\gamma M^2 - (\gamma-1) - (\gamma+1)\xi}}$$

$$\times \left\{\frac{2\gamma M^2 - (\gamma-1) - (\gamma+1)\xi}{\left[(\gamma+1)\xi + (\gamma-1)\right]\left[\gamma M^2-\xi+1\right]} - \frac{(\gamma+1)\ (\xi-1)}{\left[(\gamma+1)\xi + (\gamma-1)\right]^2}\right\} \tag{5}$$

where

$$u = \frac{\xi - 1}{\gamma M^2 - \xi+1} \sqrt{\frac{2\gamma M^2 - (\gamma-1) - (\gamma+1)\xi}{(\gamma+1)\ \xi + (\gamma-1)}} \tag{6}$$

Equation 5, when written for both streams and inserted into Equation 4, yields the solution for the pressure ratio at maximum shock deflection angle. The resultant equation is complex and must be solved by a numerical procedure. A first approximation for the numerical procedure is the pressure ratio for the maximum shock deflection of the weak stream. The shock wave angle for maximum deflection of a single stream is (Reference 4, Equation 168)

$$\lambda_{\beta_{MAX}} = \sin^{-1} \sqrt{\frac{1}{4\gamma M^2}\left\{(\gamma+1)M^2 - 4 + \sqrt{(\gamma+1)\left[(\gamma+1)M^4 + 8(\gamma-1)M^2+16\right]}\right\}} \tag{7}$$

and

$$\xi_{\beta_{MAX}} = \frac{2\gamma M^2 \sin^2\lambda_{\beta_{MAX}} - (\gamma-1)}{\gamma + 1} = \xi_1 \tag{8}$$

where program logic selects the weak stream and then calculates ξ_1 to initiate the calculation of the pressure ratio corresponding to the maximum total deflection. After Equation 4 is solved for $\xi_{\beta TM}$, Equation 2 is used for both streams and there results

$$\beta_{TOT_{MAX}} (\xi_{\beta TM}) = \beta_I (\xi_{\beta TM}, M_I, \gamma_I) + \beta_E (\xi_{\beta TM}, M_E, \gamma_E) \qquad (9)$$

Then, if $\beta_{TOT_{MAX}} \rightarrow \Delta\theta$, a solution is possible. When a solution is possible, it is found by rewriting Equation 1 in the form:

$$F(\xi) = 0 = \beta_I(\xi, M_I, \gamma_I) + \beta_E(\xi, M_E, \gamma_E) - \Delta\theta \qquad (10)$$

and the value of ξ which satisfies this equation can be found by the Newton root method using Equations 4, 5, and 6 to calculate the derivative of the function $F(\xi)$. Once the solution for Equation 10 is found, the slipline angle may be calculated with either of the following equations.

$$\theta_S = \theta_{BI} - \beta_I (\xi, M_I, \gamma_I) \qquad (11)$$

$$\theta_S = \theta_{BE} + \beta_E (\xi, M_E, \gamma_E) \qquad (12)$$

The above procedures were coded into a subroutine compatible with the base pressure program, checked out with a small driver routine and then incorporated in the program in place of the original slipline subroutine. Logically, the subroutine must work for either stream, the weak/strong stream and for equal strength streams, i.e., β_{IM} and β_{EM} are found at a single value of ξ. In the case of equal strength streams, $\beta_{TOT_{MAX}}$ is given directly by

$$\beta_{TOT_{MAX}} = \beta_{I_{MAX}} + \beta_{E_{MAX}} \qquad (13)$$

both of which are calculated in the process to identify which of the two streams are the weak and strong streams. The overall performance of a Newton root method for solving an equation such as Equation 10 often depends upon the adequacy of the first approximation for the independent variable. Since β_{T1} and ξ_1 are known from the solution for $\beta_{TOT_{MAX}}$, they are used to assign a first approximation value of the pressure ratio, for example, ξ_a. If a solution is possible and $\Delta\theta \geq \beta_{T1}$, then set $\xi_a = \xi_1$. However, if $\Delta\theta < \beta_{T1}$, then

$$\xi_a = 1 + \frac{\Delta\theta}{\beta_{T1}} (\xi_1 - 1) \tag{14}$$

and with these first approximations, the subroutine consistently obtained solutions in either range with only five or six iterations.

All possible logic situations for the new slipline subroutine were exercised and checked out with the driver program using designed input data. The subroutine will handle all conceivable relations between the two streams and in the computation of a large number of cases there was not a single failure to meet the required iteration tolerance in five or six iterations. After these exercises, the new slipline subroutine was placed in the base pressure program superceding the original one and some critical case calculations were performed. Table 3 presents results calculated for exactly the same case as that of Table 2.

Inspection of these two tables shows a minor difference in the base pressure ratio of the two solutions. For this critical case, the two subroutines follow a different path through the iteration procedure to the base pressure solution. Comparing the values of BLDR for the two solutions, it is seen that the strong shock version reaches the tolerance in the iteration with a greater residual than the weak shock version. This is caused by the ability of strong shock subroutine to find slipline angles for base pressure values that the weak shock solution cannot find. Thus, the iteration paths differ. Both versions of the program were run for this geometry at a Mach number of 1.4508 and the results in this case were identical.

The solution at a free stream Mach number of 1.4308 (Tables 1 and 2) was the lowest free stream Mach number solution found by the CYBER and Perkins-Elmer program with the weak shock solution for the slipline. With the strong shock slipline solution, base pressure solutions were found to a free stream Mach number of 1.3908 for this same critical case geometry and pressure ratio. This is a significant improvement in overall critical case performance of the program. Results of this calculation are presented in Table 4. A FORTRAN listing of the new slipline subroutine is given in Appendix C.

IV. BOUNDARY LAYER EFFECTS

Boundary layer effects upon power on base pressure have, in one method of approach (References 5 through 10), been included by assuming that the boundary layer at the point of separation may be converted to an equivalent shear layer which originates upstream of the actual separation point. In this approach, the usual assumption is that there is a fully developed velocity profile in the shear layer both at the point of separation and downstream where the two streams interact. The major areas to be resolved, then, are the effects of the upstream boundary layer upon the

TABLE 4. CRITICAL CASE PERKIN-ELMER 3230 SOLUTION
STRONG SHOCK SLIPLINE SUBROUTINE
M = 1.3908

```
       ****TWO-STREAM BASE PRESSURE PROGRAM****        PROB. NO.    1
******INPUT DATA******

                           ***INTERNAL STREAM***

GAMMAI= 1.400                GAS CONSTANT =      55.16 LB-FT/LB-R
X1I=   0.000                 R1I=    0.250          BETA1I(DEG)= 10.000
EMN1I  = 2.7000              EMS1I = 1.8865         P1I/POI = 0.04295

                           ***EXTERNAL STREAM***

GAMMAE=  1.400               GAS CONSTANT =       53.35 LB-FT/LB-R
X1E=  0.000                  R1E=  1.250            BETA1E(DEG)=  0.000
EMN1E = 1.3908               EMS1E = 1.2937         P1E/PO1E = 0.31833

                      ******BASE PRESSURE CASE DATA******


P1I/PE =  2.1529                                    TOE/TOI = 1.00000
BLDRO  = 0.00000E+00                                ENGRO = 0.00000E+00

                  ***RECOMPRESSION COEFFICIENT = 0.668***
                  *******************************************

                        ***ISOENERGETIC SOLUTION***
                        ***************************

                  ******TURBULENT JET MIXING RESULTS******

                           ***CURRENT DATA***
P1I/PE   =  2.15291                                 TOE/TOI =  1.00000
POE/POI  =  0.06267                                 POI/PE  =  50.126
BLDRO    =  0.00000E+00                             ENGRO = 0.00000E+00

                           ***MIXING DATA***

BLDR = -0.47344E-03                                 ENGR = 0.48273E-04

               ***BASE PRESSURE AND TEMPERATURE RESULTS***
TB/TOE =  1.00000                                   TB/TOI  =  1.00000
PB/PE  =  0.33764                                   PB/P1I  =  0.15683
CP-B   = -0.48918                                   CD-B    =  0.46961

                 ******END OF CURRENT CASE RESULTS******
                 ***************************************
```

downstream mixing equations and a procedure for calculating the origin shift for the equivalent mixing length. In the following sections, the mixing region equations are derived assuming that the origin shift, X_o, is defined and then relationships for defining the origin shift are presented.

A. Two-Dimensional Constant-Pressure Mixing Region

The analysis for the mixing region follows closely the procedure of Reference 11. Since this reference presents in detail the derivation of the mixing equations without upstream boundary layer terms, the derivation of two critical equations will be presented in detail and the remaining will be summarized. The control volume used to derive the two-dimensional mixing equations is shown in Figure 9 where the boundary layer profile on the upstream edge of the control volume is assumed to be the profile after the expansion at the separation point. Also shown in this figure, is the concept of an origin shift, X_o, for the shear layer mixing profile on the downstream edge of the control volume. Therefore, this shear layer has an effective mixing length of $X_o + X$.

This derivation is for a constant pressure mixing region with (1) a turbulent boundary layer velocity profile at the separation point with the values of δ, δ^* and $\hat{\theta}$ [1] known from the upstream boundary layer solution, and (2) the assumption of an equivalent fully developed mixing layer profile both at the separation point and at the downstream location. The velocity profile within the mixing zone originally proposed by Korst and used by Addy for the base pressure program is

$$\phi \equiv \frac{U}{U_a} = \frac{1}{2}\left[1 + \mathrm{erf}(\eta)\right] \tag{15}$$

and for the present application.

$$\eta = \frac{\sigma y}{x_o + x} \tag{16}$$

where (x, y) refer to the intrinsic coordinate system and σ is the similarity parameter.

The intrinsic coordinate system is located relative to the reference coordinate system by applying the momentum equation in the X-direction (per unit width) to the control volume of Figure 9. Accounting for the boundary layer profile, the momentum equation is

---

1. δ, δ^* and $\hat{\theta}$ are, respectively, the total thickness, displacement thickness and momentum thickness of the upstream boundary layer after the corner expansion.

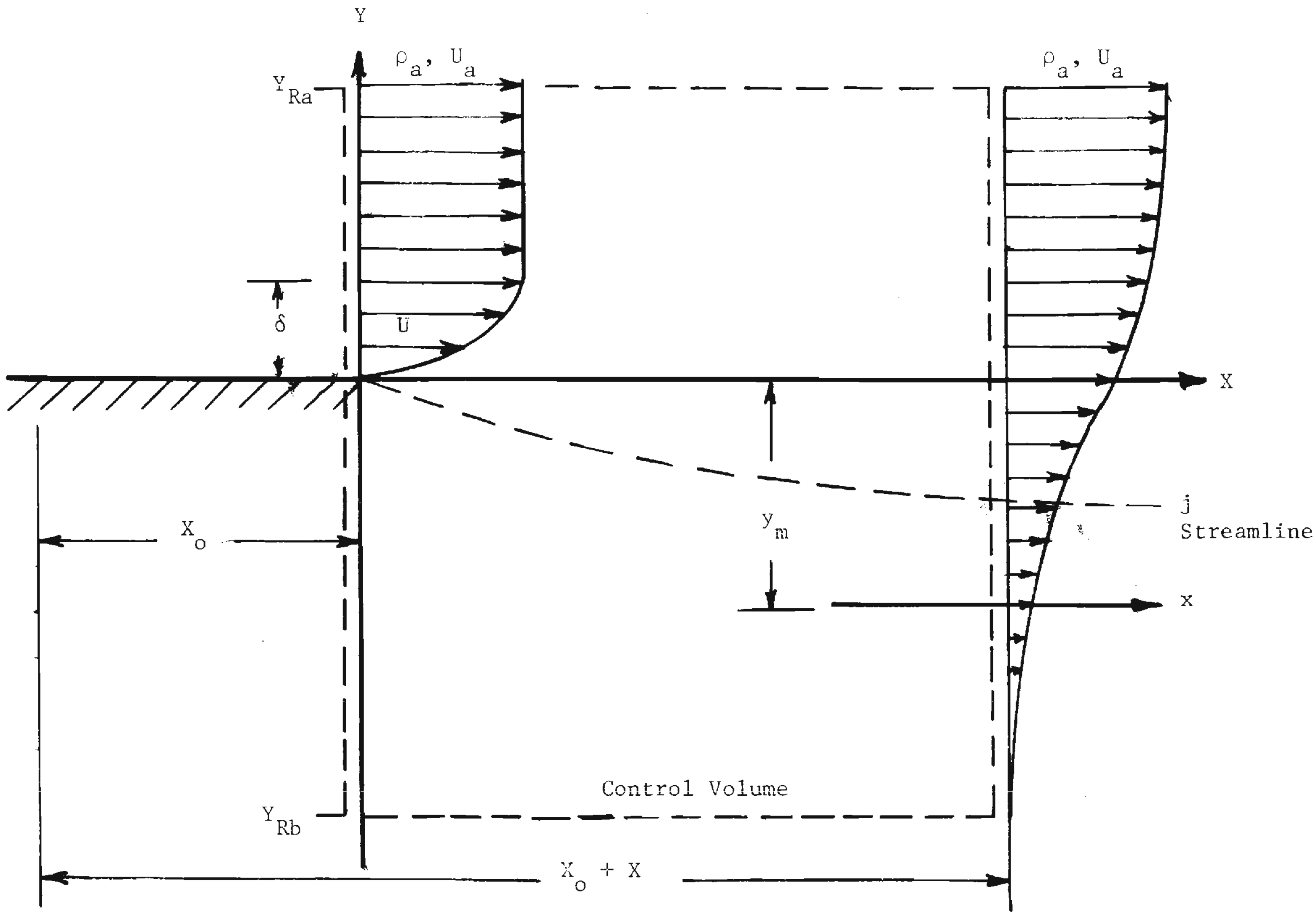

Figure 9. Control volume for mixing analysis.

$$-\rho_a U_a^{\,2}(Y_{Ra}-\delta) - \int_0^{\delta} \rho U^2 dy + \int_{Y_{Rb}}^{Y_{Ra}} \rho U^2 dy = 0 \qquad (17)$$

now, the relationship between the reference and intrinsic coordinates is

$$y = y_m(X) + Y \qquad\qquad x_o + x \approx X_o + X \qquad (18)$$

From Equations 16 and 18

$$y = \frac{(x_o+x)\eta}{\sigma} \; ; \quad dy = \frac{(x_o+x)}{\sigma} d\eta \; ; \quad dY = dy \qquad (19)$$

While from standard boundary layer defintions

$$\int_0^{\delta} \rho U^2 dy = \rho_a U_a^{\,2}(\delta-\delta^{*}-\hat{\theta}) \qquad (20)$$

When these results are substituted into Equation 17, then

$$-\rho_a U_a^{\,2}(Y_{Ra}-\delta) - \rho_a U_a^{\,2}(\delta-\delta*-\hat{\theta}) + \int_{y_m + y_{Rb}}^{y_m + y_{Ra}} \rho U^2 dy = 0 \qquad (21)$$

When this equation is non-dimensionalized with (ρ_a, U_a) and the integration variable is changed to η

$$-(Y_{Ra} - \delta + \delta - \delta* - \hat{\theta}) + \int_{\eta_m+\eta_{Rb}}^{\eta_m+\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 \frac{(x_o + x)}{\sigma} d\eta = 0 \qquad (22)$$

then

$$-\frac{Y_{Ra}\sigma}{x_o+x} + \frac{\delta*\sigma}{x_o+x} + \frac{\hat{\theta}\sigma}{x_o+x} + \int_{\eta_m+\eta_{Rb}}^{\eta_m+\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 d\eta = 0 \qquad (23)$$

The value of Y_{Rb}, and thus η_{Rb}, is chosen so that ϕ=0 when Y is less than Y_{Rb} and Y_{Ra} is chosen so that $\phi = \rho/\rho_a = 1$ when Y is greater than Y_{Ra}. Therefore, when Equation 23 is expanded,

$$-\eta_{Ra} + \frac{\delta^*\sigma}{x_o+x} + \frac{\hat{\theta}\sigma}{x_o+x} + \int_{\eta_m+\eta_{Rb}}^{\eta_{Rb}} \frac{\rho}{\rho_a}\phi^2 d\eta + \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi^2 d\eta + \int_{\eta_{Ra}}^{\eta_{Ra}+\eta_m} \frac{\rho}{\rho_a}\phi^2 d\eta = 0 \qquad (24)$$

the integrand of the first integral is zero and the integrand of the third integral is one and the resultant equation is

$$-\eta_{Ra} + \frac{\delta^*\sigma}{x_o+x} + \frac{\hat{\theta}\sigma}{x_o+x} + \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi^2 d\eta + \eta_m = 0 \qquad (25)$$

Solving this equation for the location of the intrinsic system with respect to the reference system, there results

$$\eta_m = \eta_{Ra} - \frac{\delta^*\sigma}{x_o+x} - \frac{\hat{\theta}\sigma}{x_o+x} - \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi^2 d\eta \qquad (26)$$

One of the crucial elements of the mixing region is the identification of the stream line in the shear layer which divides the shear layer mass flow into the mass flow in the free stream (including the boundary layer) approaching the point of separation from the base mass flow entrained by the shear layer. This streamline is usually called the dividing or j-streamline. Application of the continuity equation to the flow on the boundaries of the control volume yields

$$-\rho_a U_a (Y_{Ra}-\delta) - \int_0^{\delta} \rho U dy + \int_{Y_j}^{Y_{Ra}} \rho U dY = 0 \qquad (27)$$

Again, using Equations 16, 18, and 19 along with the standard boundary layer definition

$$\delta - \delta^* = \int_0^{\delta} \rho U dy \tag{28}$$

Equation 27 reduces to

$$\frac{-Y_{Ra}\sigma}{x_o+x} + \frac{\delta^*\sigma}{x_o+x} + \int_{\eta_j}^{\eta_{Ra}+\eta_m} \frac{\rho}{\rho_a} \phi d\eta = 0 \tag{29}$$

and the integral is expanded into

$$-\eta_{Ra} + \frac{\delta^*\sigma}{x_o+x} + \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi d\eta + \int_{\eta_{Ra}}^{\eta_{Ra}+\eta_m} \frac{\rho}{\rho_a} \phi d\eta = 0 \tag{30}$$

which becomes

$$-\eta_{Ra} + \frac{\delta^*\sigma}{x_o+x} + \eta_m + \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi d\eta = 0 \tag{31}$$

Now Equation 26 is substituted into Equation 31 for η_m resulting in

$$-\eta_{Ra} + \frac{\delta^*\sigma}{x_o+x} + \eta_{Ra} - \frac{\delta^*\sigma}{x_o+x} - \frac{\hat{\theta}\sigma}{x_o+x} - \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 d\eta + \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi d\eta = 0 \tag{32}$$

which becomes

$$\int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi d\eta = \frac{\hat{\theta}\sigma}{x_o+x} + \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 d\eta \tag{33}$$

Equation 33 is the integral equation which must be solved for the location, η_j, of the dividing streamline in the solution of the mixing equations. The analysis of Reference 11 for the case of flow without a boundary layer shows the location of the dividing streamline to be independent of location along the shear layer length. However, Equation 33 shows that when the upstream boundary layer is included in the analysis, the location of the dividing streamline varies along the shear layer length.

The remaining equations derived for the mixing region differ from those derived in Reference 11 only by the difference in mixing layer lengths produced by the effective origin shift. Due to this similarity, the derivation will not be repeated but the remaining equations are summarized below. The energy transferred across the j-streamline is

$$\frac{\sigma}{x_o+x} \frac{e_j}{\rho_a U_a C_p T_{oa}} = \int_{\eta_{Rb}}^{\eta_j} (\Lambda - \Lambda_B) \frac{\rho}{\rho_a} \phi d\eta \tag{34}$$

Another streamline of critical interest in the overall problem is the streamline which stagnates at the recompression shock. This streamline is sometimes called the discriminating, or d, streamline. Energy convected by the mass flux between the j and d streamlines is given by

$$\frac{\sigma}{x_o+x} \frac{e_d}{\rho_a U_a C_p T_{oa}} = \int_{\eta_{Rb}}^{\eta_d} \frac{\rho}{\rho_a} \Lambda\phi d\eta - \int_{\eta_{Rb}}^{\eta_j} \frac{\rho}{\rho_a} \Lambda\phi d\eta \tag{35}$$

The total rate of energy transfer per unit width to the wake is found by combining Equations 34 and 35. After some simplification, the result is

$$\frac{\sigma}{x_o+x}\ \frac{e}{\rho_a U_a C_p T_{oa}} = -\Lambda_B \int_{\eta_{Rb}}^{\eta_j} \frac{\rho}{\rho_a}\ \phi d\eta + \int_{\eta_{Rb}}^{\eta_d} \frac{\rho}{\rho_a} \Lambda \phi d\eta \tag{36}$$

Finally, the mass flow rate which is convected between the j and d streamlines per unit width is given by

$$\frac{\sigma}{x_o+x}\ \frac{g_d}{\rho_a U_a} = \int_{\eta_{Rb}}^{\eta_d} \frac{\rho}{\rho_a}\ \phi d\eta - \int_{\eta_{Rb}}^{\eta_j} \frac{\rho}{\rho_a}\ \phi d\eta \tag{37}$$

Equations 33, 36, and 37 define results for the mixing analysis after ϕ and ρ/ρ_a are determined. The origin shift, x_o, which determines the effective mixing length is determined by the analysis in the following section.

To incorporate these equations into the base pressure program, they must be non-dimensionalized and generalized to the axi-symmetric configuration. The procedures for these steps are identical to those of Reference 11, which are given in great detail, and they will not be repeated herein. These steps have been completed and the mixing layer equations containing the boundary layer terms have been included in a revised version of the base pressure program. Some initial results from this revised program will be presented and discussed later in this section.

B. Origin Shift

As discussed above, the preferred method for including the upstream boundary layer is to equate a boundary layer parameter to an equivalent free shear layer parameter and use this relationship to calculate the virtual origin of the free shear layer. Reference 10 originally proposed to match the entire momentum defect of the free shear layer to the momentum thickness of the boundary layer. This approach was summarized in Reference 6 and is presented below. The boundary layer momentum thickness is defined by

$$\hat{\theta} = \int_o^{\delta} \frac{\rho U}{\rho_a U_a}\ \left(1 - \frac{U}{U_a}\right) dy \tag{38}$$

If this equation is written in terms of the shear layer variables and integrated over the entire height of the shear layer, there results

$$\hat{\theta} = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi(1-\phi) \frac{x}{\sigma} d\eta \tag{39}$$

or

$$\frac{\sigma\hat{\theta}}{x_o} = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi(1-\phi)\ d\eta \tag{40}$$

To gain an insight into the physical significance of this result, rewrite Equation 40 in the form

$$\frac{\sigma\hat{\theta}}{x_o} = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi d\eta - \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 d\eta \tag{41}$$

For the case of no upstream boundary layer, the j-streamline equation gives

$$\int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 d\eta = \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi d\eta \tag{42}$$

After Equation 42 is substituted into Equation 44 and some simplification is done to the resultant and further including Equation 40

$$\frac{\sigma\hat{\theta}}{x_o} = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi(1-\phi) d\eta = \int_{\eta_{Rb}}^{\eta_j} \frac{\rho}{\rho_a} \phi d\eta \tag{43}$$

This last form of the equation shows the origin shift term to be equivalent to a bleed mass flow which is equivalent to the entire mass flow below the j-stream line at the separation point.

A different approach to the virtual origin displacement is given in Reference 8 and is well confirmed by experimental data. This approach is based on the relationship

$$\frac{\sigma\hat{\theta}}{x_o} = \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi(1-\phi)\, d\eta \tag{44}$$

which is obtained by equating the momentum defect of the shear layer above the dividing streamline to the boundary layer momentum thickness. When Equation 44 is split into two integrals and combined with Equation 42, the result is

$$\frac{\sigma\hat{\theta}}{x_o} = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 d\eta - \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi^2 d\eta \tag{45}$$

Splitting the first integral into two integrals by changing the limits of integration, and using Equation 40

$$\frac{\sigma\hat{\theta}}{x_o} = \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a} \phi(1-\phi)\, d\eta = \int_{\eta_{Rb}}^{\eta_j} \frac{\rho}{\rho_a} \phi^2 d\eta \tag{46}$$

Equation 46 demonstrates that the momentum defect above the j-streamline is equal to the momentum of the shear layer below the j-streamline and this is matched to the boundary layer momentum thickness. Based upon the correlation with experimental data, Equation 46 has been chosen for implementation in the base pressure program.

C. Corner Expansion Effect on Boundary Layer Momentum Thickness

The boundary layer momentum thickness in the equations of the section above must be the one which exists after the boundary layer traverses the corner expansion flow field. A number of authors (References 7 and 12 through 15) have proposed solutions for the corner expansion effect on boundary layer momentum thickness. White (Reference 13) compared results from his analysis with results from the analyses of References 7 and 12. This comparison showed results from these analyses to be very similar. Further, there are no data available which would indicate the choice of a particular one of these analyses. The analysis of Reference 7 was chosen for application to the base pressure program since it is one of the easiest to implement. Equation 3.10 of Reference 7 is

$$\frac{\rho_B \, U_B \, \hat{\theta}_B}{\rho_{1a} \, U_{1a} \, \hat{\theta}_1} = \frac{M_{1a}^{2}}{M_{Ba}^{2}} \tag{47}$$

and for application to the computer program it is transformed into

$$\frac{\hat{\theta}_B}{\hat{\theta}_1} = \frac{M_{1a}^{3}}{M_{Ba}^{3}} \left[\frac{1 + \frac{\gamma-1}{2} M_{Ba}^{2}}{1 + \frac{\gamma-1}{2} M_{1a}^{2}} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{48}$$

which completes the overall set of equations for incorporating the upstream boundary layer effects into the base pressure program.

D. Discussion and Results

Results from the analyses presented above were formulated for both external and internal streams and were incorporated into the Perkin-Elmer version of the base pressure computer program. These modifications to the computer program were thoroughly verified and a set of solutions was obtained for one configuration. Calculations for this configuration were presented in Reference 6 for several different combinations of origin shifts, boundary layer terms and recompression criteria. The current solutions, with and without the boundary layer effects, were compared with three selected solutions and the experimental data from Figure 6 of Reference 6. This comparison is presented in Figure 10. It is seen that the two versions of the program with no boundary layer effects and the empirical recompression criteria developed by Addy in Reference 3 yield identical results. These results were somewhat lower than the experimental data of Reference 16 but were probably within the expected range considering the deviation of the correlation results for this recompression criteria. Also, included in this figure are the Reference 6 calculations using the ONERA shift calculations and the ONERA angular recompression criterion. This particular solution was the most appropriate one to compare with the current solution which was boundary layer effects and uses the empirical correlation for the recompression criterion. The general agreement of these two calculations was reasonably good, but both were somewhat higher than the experimental data. Over prediction of experimental data by the current calculations was to be expected since the empirical recompression criterion resulted from a correlation of experimental data, with boundary layer effects, using the original version of the base pressure program which did not include boundary layer terms in the solution. The third solution from Reference 6 shown in Figure 10 was one using the modified ONERA criterion for the recompression and the ONERA origin shift calculations and this calculation showed excellent agreement with the experiment data.

This comparison shows that the effect of the boundary layer upon the overall solution is very significant. However, the recompression criteria also was a large factor in the overall solutions. On the basis of the results of Reference 6, the modified ONERA criterion for the recompression was the best choice; however, more comparisons for experimental data and calculations should be made before a final selection is made for a recompression criterion.

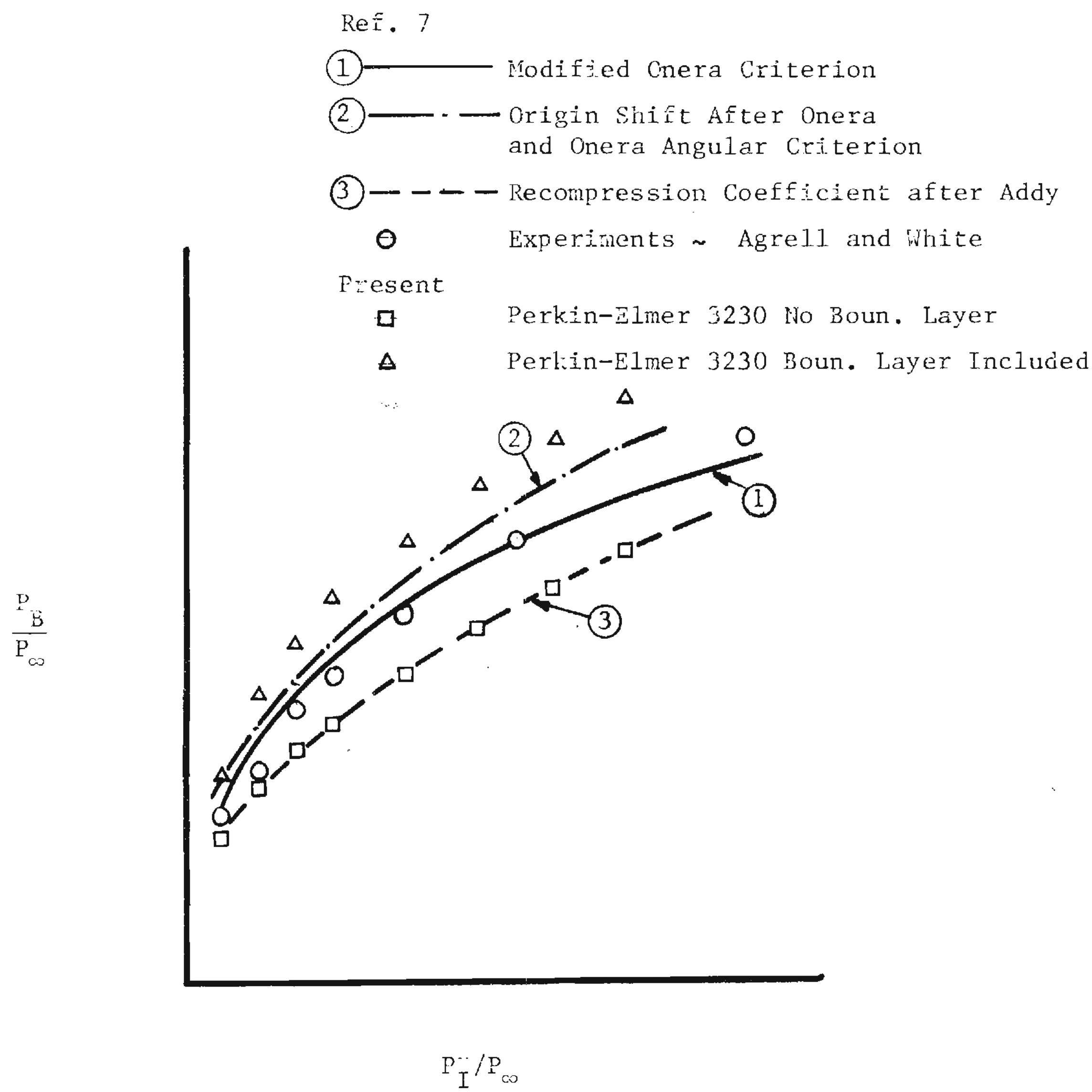

Figure 10. Comparison of various P_B calculations for cylindrical afterbody with experimental data.

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APPENDIX A

DEFINITION OF PERKIN-ELMER INTERACTIVE SCREEN CUES AND INPUT DATA

APPENDIX A

```
CUE:        ENTER ALPHANUMERIC HEADING - 20A4
            READ (5,10) (A(I),I=1,20)
            FORMAT (20A4)
            A(I) = ANY ALPHANUMERIC PROBLEM DESCRIPTION

            AFTERBODY SHAPE PARAMETERS
              0 - CYLINDRICAL AFTERBODY
              1 - OGIVE BOATTAIL
CUE:          2 - PARABOLIC BOATTAIL
              3 - CONICAL BOATTAIL OR FLARE
            ENTER AFTERBODY SHAPE PARAMETER - I1
            READ (5,13) NSHAPE
            FORMAT(I1)
            NSHAPE = PARAMETER DEFINING THE AFTERBODY SHAPE

            BODY AND NOZZLE DIMENSIONS ARE RELATIVE
MESSAGE:    THEY CAN BE INCHES, FEET, OR CALIBERS
            "X" DIMENSIONS ARE POSITIVE AFT

CUE:        ENTER "X" AT BODY BASE
            READ(5,11) X1E
            FORMAT (4F10.4)
            X1E = LONGITUDINAL COORDINATE OF POINT WHERE SEPARATION OF THE
                  EXTERNAL STREAM OCCURS

CUE:        ENTER RADIUS AT BODY BASE
            READ(5,11) R1E
            FORMAT (4F10.4)
            R1E = RADIAL COORDINATE OF POINT WHERE SEPARATION OF THE
                  EXTERNAL STREAM OCCURS
```

The following three cues are dependent upon the value of NSHAPE. If NSHAPE = 1, 2, or 3, these cues appear. If NSHAPE = 0, they do not appear and the cue for the free stream Mach number appears.

```
CUE:        ENTER "X" AT START OF BOATTAIL
            READ(5,11) X2E
            FORMAT (4F10.4)
            X2E = INITIAL LONGITUDINAL COORDINATE OF THE BOATTAIL

CUE:        ENTER RADIUS AT START OF BOATTAIL
            READ (5,11) R2E
            FORMAT (4F10.4)
            R2E = INITIAL RADIAL COORDINATE OF THE BOATTAIL
```

CUE: ENTER SLOPE AT START OF BOATTAIL - DEG

READ (5,11) BETD2E

FORMAT (4F10.4)

BETD2E = INITIAL BOATTAIL ANGLE (IN DEGREES) AT (X2E, R2E) COUNTER-CLOCKWISE FROM X-AXIS IS POSITIVE.

CUE: ENTER FREE STREAM MACH NUMBER

READ (5,11) EMNE

FORMAT (4F10.4)

EMNE = EXTERNAL FREE STREAM MACH NUMBER

CUE: ENTER "X" AT END OF NOZZLE

READ (5,11) X1I

FORMAT (4F10.4)

X1I = LONGITUDINAL COORDINATE OF POINT WHERE SEPARATION OF THE INTERNAL STREAM OCCURS

CUE: ENTER NOZZLE EXIT RADIUS

READ (5,11) R1I

FORMAT (4F10.4)

R1I = RADIAL COORDINATE OF POINT WHERE SEPARATION OF THE INTERNAL STREAM OCCURS

CUE: ENTER NOZZLE EXIT ANGLE - DEG.

READ (5,11) BETD1I

FORMAT (4F10.4)

BETD1I = FLOW ANGLE (IN DEGREES) AT (X1I, R1I) COUNTER-CLOCKWISE IS POSITIVE

CUE: ENTER NOZZLE GAS CONSTANT - LBF/LBM R
53.34 FOR AIR

READ (5,11) GCI

FORMAT (4F10.4)

GCI = GAS CONSTANT FOR THE INTERNAL STREAM

CUE: ENTER NOZZLE GAMMA - 1.4 FOR AIR

READ (5,11) GAMMAI

FORMAT (4F10.4)

GAMMAI = RATIO OF SPECIFIC HEATS FOR THE INTERNAL GAS

CUE: ENTER NOZZLE EXIT MACH NUMBER

READ (5,11) EMN1I

FORMAT (4F10.4)

EMN1I = MACH NUMBER AT (X1I, R1I)

CUE: ENTER EXTERNAL TO INTERNAL STREAM STAGNATION TEMPERATURE RATIO - TOE/TOI

READ (5,11) TROEI

FORMAT (4F10.4)

TROEI = STAGNATION TEMPERATURE RATIO OF STREAMS, TOE/TOI

```
CUE:        ENTER NUMBER OF CASES - I1
            READ (5,13) NCASE
            FORMAT (I1)
            NCASE = NUMBER OF PRESSURE RATIOS, P1I/PE OR POI/PE, FOR WHICH
                    BASE PRESSURE CALCULATIONS ARE TO BE MADE FOR A GIVEN
                    SET OF CONDITIONS AND GEOMETRY

            ENTER TYPE OF PRESSURE RATIO INPUT
CUE:          0 FOR INTERNAL STATIC/EXTERNAL STATIC PRESSURE
              1 FOR INTERNAL STAGNATION/EXT STATIC PRESSURE
            READ (5,13) KPRESR
            FORMAT (I1)
            KPRESR = 0, PR1IE (P1I/PE) IS INPUT, AND PROIE IS CALCULATED.
                   = 1, PROIE (POI/PE) IS INPUT, AND PR1IE IS CALCULATED.
```

The following cues depend upon the value of KPRESR. If KPRESR = 0, then

```
CUE:        ENTER PJ/PF
            READ (5,11) PRATIO
            FORMAT (4F10.4)
            PRATIO = PRESSURE RATIO (P1I/PE = PR1IE) FOR CASE 1.
```

If KPRESR = 1, then

```
CUE:        ENTER POJ/PF
            READ (5,11) PRATIO
            FORMAT (4F10.4)
            PRATIO = PRESSURE RATIO (POI/PE = PROIE) FOR CASE 1.
```

The cues PJ/PF or POJ/PF reappear after the solution for each pressure ratio is obtained until the NCASE solutions have been completed. After the solution for the final pressure ratio is completed, the first cue returns and a complete new case may be entered.

Cues are typed above exactly as they appear on the screen and in the order in which they appear. For the input data which are real numbers, a format cue in the form of XXXXX.XXXX also comes on the screen. Nmemonics for the input data are the ones used in Reference 2 and the definitions are basically the same as presented in Reference 2.

APPENDIX B

DEFINITION OF SYMBOLS USED IN COMPUTER PRINT FOR CYLINDRICAL AFTERBODIES

APPENDIX B

Definitions presented in this appendix are arranged in the same order as the output and identified by the print section headers (compare with Tables 1 and 2).

INTERNAL STREAM

GAMMAI	= Ratio of specific heats, C_p/C_v, of the nozzle gas.
X1I	= Longitudinal coordinate of nozzle exit plane which is also the point of separation for the internal stream.
R1I	= Radial coordinate of nozzle wall at the exit plane.
BETA1I	= Nozzle wall angle at exit plane. Must be zero or positive. Positive is counter-clockwise with respect to the X-axis.
EMN1I	= Mach number of the internal (nozzle) flow at the nozzle lip (X1I, R1I).
EMS1I	= Mach Star, V/C*, of the internal flow at the nozzle lip.
P1I/P0I	= Ratio of static pressure at nozzle lip to nozzle total (chamber) pressure.

EXTERNAL STREAM

GAMMAE	= Ratio of specific heats of the external flow gas.
X1E	= Longitudinal coordinate of the missile base, which is also the point of separation for the external stream.
R1E	= Radial coordinate of the missile base.
BETA1E	= Angle of missile afterbody at base (X1E, R1E)
EMN1E	= Mach number of external flow at missile base (Free stream Mach number for cylindrical afterbody. Computed in the program for non-cylindrical afterbody).
EMS1E	= Mach Star, V/C*, of the external flow at the missile base.
P1E/P01E	= Ratio of external stream static pressure at the missile base to the external stream total pressure.

******BASE PRESSURE CASE DATA******

P1I/PE = Ratio of nozzle lip (internal flow) static pressure to free stream static pressure.

TOE/TOI = Ratio of external stream total temperature to internal stream total temperature.

BLDRO = Specified value of the base bleed ratio.

ENGRO = Specified value of the added base energy ratio. <u>Note:</u> In the current Perkin-Elmer version of the program, BLDRO and ENGRO are set equal to zero in the program.

The quantities defined above are either input data or computed in the boattail flow field calculations which precede the base pressure calculation. The subsequent sections present some of the input data which is passed through the program and solution data of interest. Where a quantity is printed again in this section, the definition is not repeated.

POE/POI = Ratio of stagnation pressure of the external stream to the stagnation pressure of the internal stream

POI/PE = Ratio of the stagnation pressure of the internal stream to the free stream static pressure.

BLDR = Mass bleed ratio, referenced to internal stream

ENGR = Energy bleed ratio, referenced to 0° and the energy of the internal stream. <u>Note:</u> The solution P_B, T_B is found to drive to an equilibrium solution where BLDR = BLDRO $\pm$ E1 and ENGR = ENGRO $\pm$ E2. Thus, these two quantities allow an assessment of the overall solution.

TB/TOE = Ratio of base gas temperature to external stream total temperature.

TB/TOI = Ratio of base gas temperature to internal stream total temperature.

PB/PE = Ratio of base pressure to free stream static pressure

PB/P1I = Ratio of base pressure to nozzle lip static pressure

CP-B = Base pressure coefficient, $(P_B - P_\infty)/q_\infty$

CD-B = Base drag coefficient, $-C_P[1 - (R_{1I}/R_{2E})^2]$

APPENDIX C

FORTRAN LISTING OF THE STRONG SHOCK SOLUTION SUBROUTINE

APPENDIX C

```
      SUBROUTINE SLIP(EMS1,THETA1,GAMMA1,EMS2,THETA2,GAMMA2,
     1 THETAS,NSIDE)
C
C     THIS SUBROUTINE CALCULATES THE SLIPLINE ANGLE FOR THE
C     OBLIQUE SHOCK RECOMPRESSION SYSTEM WHICH OCCURS AT THE
C     IMPINGEMENT POINT OF TWO SUPERSONIC STREAMS IF IT EXISTS.
C     ONE OF THE SHOCKS MAY BE A STRONG SHOCK.
C
C     SUBROUTINE REQUIRED----SISTM
C
C     ****VARIABLES****
C
C     EMS1   = MACH STAR OF STREAM 1.
C     THETA1 = FLOW ANGLE OF STREAM 1.
C     GAMMA1 = RATIO OF SPECIFIC HEATS FOR STREAM 1.
C     DELT1  = SHOCK TURNING ANGLE FOR STREAM 1.
C     DDELT1 = TURNING ANGLE DERIVATIVE OF STREAM 1.
C     EMS2   = MACH STAR OF STREAM 2.
C     THETA2 = FLOW ANGLE OF STREAM 2.
C     GAMMA2 = RATIO OF SPECIFIC HEATS FOR STREAM 2.
C     DELT2  = SHOCK TURNING ANGLE FOR STREAM 2.
C     DDELT2 = TURNING ANGLE DERIVATIVE OF STREAM 2.
C     THETAS = SLIPLINE ANGLE
C     NSIDE  = 1, FOR A SOLUTION
C            = 3, FOR NO SOLUTION
C
C     NOTE THAT THETA1 IS ASSUMED LARGER THAN THETA2.
C
```

```
      EMNPSF(EMS,GAMMA)=SQRT(((2.0/(GAMMA+1.0))*(EMS**2)/
     1(1.0-((GAMMA-1.0)/(GAMMA+1.0))*(EMS**2)))
C
C     CALCULATION OF THE MAXIMUM TURNING ANGLE FOR A GIVEN
C     APPROACH MACH NUMBER AND GAMMA; NACA R-1135,EQ.168.
C
      SINWA2(EMN,GAMMA)=(0.25/(GAMMA*(EMN**2)))*((GAMMA+1.0)*
     1(EMN**2)-4.0+SQRT((GAMMA+1.0)*((GAMMA+1.0)*(EMN**4)+8.0*
     2(GAMMA-1.0)*(EMN**2)+16.0)))
C
C     SINWA2 CALCULATES THE SINE OF THE SHOCK WAVE ANGLE SQUARED
C     FOR MAXIMUM STREAM DEFLECTION BEHIND THE SHOCK (EQN 168)
C
      DELTAM(EMN,GAMMA,SIN2WA)=ATAN ((2.0*SQRT((1.0-SIN2WA)/SIN2WA)
     1*((EMN**2)*SIN2WA-1.0))/(2.0+(EMN**2)*(GAMMA+1.0-2.0*SIN2WA)))
C
C     DELTAM CALCULATES THE MAXIMUM TURNING ANGLE GIVEN THE
C     APPROACH MACH NUMBER GAMMA AND THE SINE SQUARED OF THE WAVE
C     ANGLE,SIN2WA,FOR THE MAXIMUM DEFLECTION (EQN 139A).
C
      PROSHK(EMN,SIN2WA,GAMMA)=(2.0*GAMMA*(EMN**2)*SIN2WA-GAMMA
     1 +1.0)/(GAMMA+1.0)
C
C     PROSHK CALCULATES THE STATIC PRESSURE RISE FOR AN OBLIQUE
C     SHOCK GIVEN THE APPROACH MACH NUMBER, THE SINE SQUARED OF
C     THE WAVE ANGLE AND GAMMA (EQN 128)
C
      NIT=0
      NITMAX = 15
      EMN1=EMNPSF(EMS1,GAMMA1)
      DELT1M=DELTAM(EMN1,GAMMA1,SINWA2(EMN1,GAMMA1))
      PAMAX1= PROSHK(EMN1,SINWA2(EMN1,GAMMA1),GAMMA1)
      EMN2 = EMNPSF(EMS2,GAMMA2)
      DELT2M = DELTAM(EMN2,GAMMA2,SINWA2(EMN2,GAMMA2))
```

```
      PRMAX2 = PRDSHK(EM2,SIGMA2(EM2,GAMMA2),GAMMA2)
      IF((THETA1-THETA2) .GT. (DELT1M+DELT2M)) GO TO 500
C
C     DETERMINE WHICH STREAM IS THE WEAK STREAM
C
      IF(PRMAX1 - PRMAX2)100,100,110
C
C     STREAM 1 IS THE WEAK STREAM
C
  100 PRSSA = PRMAX1
      PR1   = PRSSA
      DEL1  = DELT1M
      PRES  = (2.0*GAMMA1*EM1**2 - GAMMA1 + 1.0)/(GAMMA1+1.0)
      GO TO 120
C      STREAM  2 IS THE WEAKER STREAM
C
  110 PRSSA = PRMAX2
      PR1   = PRSSA
      DEL1  = DELT2M
      PRES  = (2.0*GAMMA2*EM2**2-GAMMA2+1.0)/(GAMMA2+1.0)
C
C      CALCULATE THE MAXIMUM TOTAL STREAM TURNING ANGLE BY A
C      NUMERICAL INTERACTION PROCEDURE
C
 120  ICOUNT = 1
      CALL STSHK(PRSSA,EM1,GAMMA1,DELT1,DDELT1)
      CALL STSHK(PRSSA,EM2,GAMMA2,DELT2,DDELT2)
      DELTA = DELT1 + DELT2
      DTHETA = THETA1 - THETA2
      DDLTA = DDELT1 + DDELT2
      FTDDEL = DDLTA
      PRSSB = PRSSA + (PRES - PRSSA)/2.0
      IF (DTHETA - DELTA) 220,150,130
```

```
C
C       WHEN SHOCK TURNING ANGLES FOR MAX. DEFLECTION OF THE NEAR
C       STREAM ARE: (A) GREATER THAN THE STREAMLINE ANGLES A WEAK
C       SHOCK SOLUTION EXISTS, (B) LESS THAN THE STREAMLINE ANGLES
C       A STRONG SOLUTION SOUGHT.
C
  130   ICOUNT =ICOUNT + 1
        CALL SIGMA(PRSSB,EMS1,GAMMA1,DELT1,DDELT1)
        CALL SIGMA(PRSSB,EMS2,GAMMA2,DELT2,DDELT2)
        F2DDEL = DDELT1 + DDELT2
        IF (ABS(F2DDEL) -1.0E-5) 150,150,140
  140   PRSSC = PRSSB-F2DDEL*(PRSSB-PRSSA)/(F2DDEL-F1DDEL)
        PRSSA = PRSSB
        PRSSB = PRSSC
        F1DDEL= F2DDEL
        IF(ICOUNT -15) 130,130,170
  150   DELMAX = DELT1 + DELT2
C
C       THIS COMPLETES THE ITERATION LOOP TO DETERMINE THE MAXIMUM
C       TURNING CAPABILITY OF THE TWO STREAMS.
C
       IF(DELMAX-DTHETA)600,160,200
  160   THETAS = THETA1 - DELT1
        NSTOP  = 1
        RETURN
C
C
  170   WRITE(7,500)ICOUNT,F2DDEL,PRSSB
  500   FORMAT('1',5X,'***CONVERGENCE ERROR IN SLIP,LOOP1    ('I3,
      1 5X,F10.0,5X,F10.6, ' ) ***')

        IF((DELT1 + DELT2 ) - DTHETA) 600,160,200
C
C       BEGIN ITERATION LOOP FOR SLIP LINE ANGLE
C
  200   NIT = 1
        IF(ABS(DDELT1+DDELT2) .LE. 1.0E-7) GO TO 150
        IF (DELTA - DTHETA) 210,210,220
```

```
210   F1ZEO = DELTA - DTHETA
      PR2 = PR1 -F1ZEO/DDELTA
      GO TO 230
220   NIT = 1
      PR1 = 1.0 + DTHETA*(PR1-1.0)/DELTA
      CALL SIGMA(PR1,MACH1,GAMMA1,DELT1,DDELT1)
      CALL SIGMA(PR1,MACH2,GAMMA2,DELT2,DDELT2)
      IF(ABS(DDELT1+DDELT2) .LE. 1.0E-6) GO TO 150
      F1ZEO = DELT1 + DELT2 - DTHETA
      PR2 = PR1 - F1ZEO/(DDELT1 + DDELT2)
230   NIT = NIT + 1
      CALL SIGMA(PR2,MACH1,GAMMA1,DELT1,DDELT1)
      CALL SIGMA(PR2,MACH2,GAMMA2,DELT2,DDELT2)
      IF(ABS(DDELT1+DDELT2) .LE. 1.0E-6) GO TO 150
      F2ZEO = DELT1 + DELT2 - DTHETA
      IF(ABS(F2ZEO) - 1.0E-5)250,250,240
240   PR2 = PR2-F2ZEO/(DDELT1 + DDELT2)
      IF(NIT - NITMAX)230,230,260
250   THETAS = THETA1 - DELT1
      NSTOP = 1
      RETURN
C
C
260   THETAS = THETA1 - DELT1
      NSTOP = 1
      WRITE(7,510) NIT,F2ZEO,PR2
510   FORMAT('1',5X,'***CONVERGENCE ERROR IN SLIP,  (',I3, 2(5X,F10.6
    1 ) , '  )***')
      RETURN
C
600   NSTOP = 3
      WRITE(7,700)
700   FORMAT(15X,44H ***SOLUTION FOR SLIPLINE ANGLE DOESN'T EXIST***
    1 //)
      RETURN
      END
      SUBROUTINE SIGMA(PRES,MACH,GAMMA,DELT,DDELT)
```

```
C
C      THIS SUBROUTINE CALCULATES THE OBLIQUE TURNING ANGLE AND
C      ITS DERIVATIVE AS A FUNCTION OF THE PRESSURE RATIO
C      ACCROSS THE SHOCK.
C
C      PRSS  = PRESURE RATIO ACROSS THE SHOCK.
C      EMN   = MACH NUMBER UPSTREAM OF THE SHOCK.
C      GAMMA = RATIO OF THE SPECIFIC HEATS.
C      DELT  = STREAMLINE TURNING ANGLE.
C      DDELT = DERIVATIVE OF THE TURNING ANGLE
C
      A=(2.0*GAMMA*EMN**2-GAMMA+1.0-(GAMMA +1.0)*PRSS)/((GAMMA
     1 +1.0)*PRSS+ GAMMA - 1.0)
      B=((PRSS - 1.0)/(GAMMA*EMN**2-PRSS +1.0))*SQRT(A)
      DELT = ATAN(B)
C
C      COMPUTATION OF THE DERIVATIVE
C
      C=A/(GAMMA*EMN**2-PRSS+1.0) - (GAMMA + 1.0)*(PRSS-1.0)/(((
     1 GAMMA +1.0)*PRSS + GAMMA - 1.0)**2)
      DDELT = (1.0/(1.0+B**2)) * ((GAMMA*EMN**2)/(GAMMA*EMN**2
     1 -PRSS+1.0))*(1.0/SQRT(A))*C
      RETURN
      END
```

NOMENCLATURE

SYMBOLS

c^*	Critical speed of sound
Cp	Specific heat at constant pressure
e	Energy transfer rate per unit width for the two-dimensional turbulent mixing region
g	Mass entrainment rate per unit width for the two-dimensional turbulent mixing region
M	Mach number
P	Absolute pressure
T	Absolute temperature
U	x-component of the velocity within the shear layer or boundary layer
u	Function defined by Equation 6
V	Magnitude of the velocity
x_o, X_o	Origin shift due to upstream boundary layer
x,y	Intrinsic coordinates in the two-dimensional mixing region
X,Y	Reference coordinates in the two-dimensional mixing region
y_m	Shift from the reference to the intrinsic coordinate system
β	Flow deflection angle across the shock wave
δ	Upstream boundary layer total thickness
δ^*	Upstream boundary layer displacement thickness
γ	Ratio of the specific heats
η	Dimensionless coordinates in the mixing region $[\sigma y/(x_o + x)]$
η_m	Dimensionless shift of the two-dimensional mixing profile

SYMBOLS (Cont'd)

ξ	Shock pressure ratio
θ	Flow angle
$\hat{\theta}$	Upstream boundary layer momentum thickness
σ	Empirical mixing parameter
ρ	Density
Λ	Stagnation temperature ratio
ϕ	Velocity ratio, (V/V_a)

SUBSCRIPTS

a	Adjacent inviscid flow; limiting location on the "positive" side of the mixing region or boundary layer
b	Adjacent quiescent region; limiting location on the "negative" side of the mixing region
B	Base region
BE	Boundary, external
BI	Boundary, internal
d	Discriminating streamline
E	External stream
I	Internal stream
j	Jet-boundary streamline
MAX	maximum
0	Stagnation conditions
SS	Strong stream

NOMENCLATURE (Continued)

SUBSCRIPTS (Cont'd)

TOT	Total
WS	Weak stream
1	Stream geometric separation point